A comprehensive yield model for Teak plantations in central Ghana

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Abstract

Ghana has a long history of teak plantation establishment, with a high concentration of teak plantations in the country's central zone. Despite this history, a locally applicable yield model has been lacking and the reference yield models available do not allow for projection of future wood flows. This paper presents a comprehensive yield model for teak plantations in central Ghana. It also allows for the projection of future wood flows, including product assortments, and provides an impetus for the development of more locally adapted yield tables. The model is based on data collected in the Akumadan plantations of Form Ghana, which were established during 2001 to 2012. The data comprise both regular permanent sample plot measurements and section measurements on trees of various sizes. Site index curves were developed and are used to classify sites within the plantations. Complemented with a standard volume equation and equations for stand volume and basal area, it forms the basis of the yield model. The addition of a taper equation and diameter distribution extends the functionality of the model, as it enables the estimation of product assortments from predicted stand characteristics with the use of a cross-cutting algorithm.

Keywords: Teak, *Tectona grandis*, Site Index, Volume equation, diameter distribution, taper equation, silvicultural model, Ghana

Introduction

Teak (*Tectona grandis* L.f.) is a tropical hardwood species of great importance and its timber is extracted from both natural forests and plantations. Teak is native to India, Myanmar, Laos and Thailand, and is naturalized in Java, Indonesia, where it was introduced about 400 to 600 years ago (Pandey & Brown, 2000; Verhaegen, et al., 2010). In the past decades, teak has become one of the most important tropical hardwood plantation species (Verhaegen, et al., 2010). Teak is highly valued for its physical and aesthetic wood properties, and the increasing demand for quality timber and declining supply from natural forests have led to an increase in the development of teak plantations (Pandey & Brown, 2000; Bermejo, et al., 2004).

Teak was first introduced in Ghana in the early 1900s (Ugalde Arias, 2013), and the current area is estimated at 180,000 ha with a high concentration in the central zone of Ghana (Brown & Kollert, 2017). Management of these plantations mostly follows simple rules of thumb, which are not site specific and do not allow optimal management. For long-term decision making and optimising forest operations, forest managers need an appropriate yield model as a decision support tool. Forest managers in central Ghana often refer to the yield tables for teak in Côte d'Ivoire (Dupuy, et al., 1999), Benin (Ganglo, 2020) and northern Ghana (Nunifu & Murchison, 1999). However, these yield models show significant deviations from the dynamics of teak growth observed in central Ghana. Moreover, these yield models do not allow for simulating harvests and the associated product mix cannot be predicted using these models. Therefore, in this study a more locally applicable yield model is presented that allows for projecting future wood flows and develop more locally adapted yield tables.

1.   Material and methods

1.1   Study area

The data for this study was collected from the Form Ghana plantations in the Asubima and the Afrenso Brohuma Forest Reserves in the Ashanti Region of Ghana. Asubima and Afrenso Brohuma Forest Reserves lie at the northern fringes of the semi-deciduous forest zone of Ghana at an altitude of approximately 300 m above mean sea level. Like most degraded forest reserves in Ghana, the study area has been affected by activities such as farming, fire damage, over-exploitation and illegal logging before the plantations were established (Abeney, et al., 2008). The zone has a tropical monsoon climate with alternating wet and dry seasons with an average annual rainfall of 1200 mm. The long rainy season (March to July) is followed by a short dry period (July/August) and a short rainy period (September/October) before the long dry season starts (November to March). Temperatures are generally high and uniform throughout the year with a mean annual temperature of 26 °C. The climate of the area is within the optimal range for teak, albeit that the annual precipitation is at the lower ranges of the optimal range for the species (Pandey, 1996). The terrain is undulating with a large ridge running from the east to the west, topped with a nearly horizontal layer of sandstone. Slopes are moderate, between 5-10% in steepness, and the hills have flat tops. Rock outcrops occur in several places in the area, but generally cover only small areas. The soil in the area have developed from weathered sandstone and generally have a sandy loam to sandy clay loam texture. Deeper horizons have a clay loam to clay texture due to illuviation of clay particles. The teak in these reserves was planted by Form Ghana over a period spanning from 2001 to 2012. The origin of the planting material is mostly Bouaké in Côte d'Ivoire, but some smaller portions have been planted with material collected from Ghana such as Pampawie Forest Reserve, Nkoranza and Jimera Clonal Seed Orchard. Other smaller areas were planted with Teak from Oumé in Côte d'Ivoire and with clones imported from Brazil (Wanders, et al., 2021). A system of selective thinning for crown liberation was applied, as is commonly applied in plantation forestry.

1.2   Data collection

1..2.1 *Section measurements*

Section measurements were done in a variety of age classes in different parts of the plantation through destructive sampling. Sections were cut with a chainsaw at regular intervals starting from the butt end up to a non-commercial diameter, which is usually 5-10 cm. For each section, the diameter was measured using a calliper, where two diameter measurements are taken at right angles to each other. The square root of the product of the two diameters is then used as the measure of stem diameter, which allows that the stem cross section may be shaped as an ellipse, rather than being circular (West, 2015).

1..2.2 *Permanent sample plots*

Data was collected from a total of 422 permanent sample plots (PSPs) ranging in ages between 3 and 19 years. The data was collected during the period 2011 to 2020, and included a total of 1809 plot measurements. Data collection in the PSPs started 3 years after planting, and was collected on a yearly basis or at 2-year intervals. The majority of the plantations were established after 2007 and most data was obtained from relatively young plantations (Table 1). As the first plantings were established in 2001, there was, however, some information available on growth in the second half of the rotation.

Table 1 Age distribution of Permanent Sample Plots

Age	# plots	Age	# plots	Age	# plots
3	328	9	307	15	7
4	15	10	6	16	8
5	254	11	98	17	9
6	119	12	7	18	9
7	322	13	7	19	18
8	288	14	7		

All permanent sample plots had a circular shape with a default radius of 15.96 m and were well distributed throughout the plantations, covering all planting years with a 1% sampling density. For each plot, all teak trees were measured for diameter at breast height and total height.

1.3 Data analysis

1.3.1. Tree volume

The volume of the individual sections of each tree were calculated using Smalian's formula. Thus, the total tree volume was calculated as the sum of the volumes of all separate sections. Table 2 shows the number of sampled trees per diameter class with the corresponding height range and computed range of tree volumes.

Table 2 Sampled trees per diameter class

Sampled trees			Ranges		
DBH class	N	%	DBH (cm)	Height (m)	Volume (m^3)
5-10	10	6.1%	7.5-9.7	6.6-12.2	0.019-0.043
10-15	103	63.2%	10.0-14.9	7.6-15.1	0.027-0.114
15-20	21	12.9%	15.0-16.9	8.9-18.6	0.074-0.169
20-25	16	9.8%	20.0-24.8	15.2-24.2	0.189-0.460
25-30	7	4.3%	25.3-29.0	19.6-23.4	0.454-0.623
30-35	2	1.2%	31.6-33.8	24.7-26.0	0.798-0.957
35-40	1	0.6%	38.5	24.8	1.039
40-45	1	0.6%	41.2	22.6	0.909
45-50	1	0.6%	48.6	25.0	1.739
50-55	1	0.6%	51.0	23.5	1.608
Total	163	100%	7.5-51.0	6.6-26.0	0.019-1.739

The total tree stem volume was estimated using the logarithmic equation (Schumacher & Hall, 1933):

$v = b_0 d^{b_1} h^{b_2}$	Eq. 1

Where, v is total tree volume to the tip of the tree (m³), d is tree diameter at breast height (cm), h is total tree height (m) and $b_0 - b_2$ are coefficients estimated by non-linear regression analysis.

1.3.2 Tree taper

In order to be able to estimate volumes of log assortments from individual trees, a tree taper function is needed. A taper function allows for estimation of both total wood volume in a tree stem and the dimensions of the individual logs that might be cut from the stem as required by the market (West, 2015). A tree taper function was fitted against the individual tree data from the section measurements. This equation has the following form (Sharma & Zhang, 2004):

$$d_i{}^2(h_i) = b_1 \times d^2 \times \left(\frac{h_i}{1.3}\right)^{2-\left(b_2 - b_3 \times \left(\frac{h_i}{h}\right) + b_4 \times \left(\frac{h_i}{h}\right)^2\right)} \times \left(\frac{h - h_i}{h - 1.3}\right)$$	Eq. 2

Where, d_i is diameter (cm) at h_i, d is diameter at breast height (cm), h_i is height (m) from ground ($0 \leq h_i \leq h$) and h is total tree height (m).

1.3.3 Site classification

Despite the fact that it is not always sufficient to characterize forest site productivity (Bontemps & Bouriaud, 2013), site index is the most commonly used indicator to evaluate forest site productivity (Burkhart & Tomé, 2012; Socha, et al., 2016). Calculation of site index is based on the stand dominant height at a certain index age. For estimating site index, the Schumacher model (Schumacher, 1939) was used, which has been used extensively in forestry for fitting anamorphic site index curves (Nanang & Nunifu, 1999). Anamorpic curves all have the same shape for a given location, but have different intercepts depending on site index. The Schumacher model has the following form:

$H = \alpha * \exp\left(\beta A^{-k}\right)$	Eq. 3

Where, H is dominant height (m), A is stand age (years), a is a coefficient that varies with site index, and b and k are coefficients estimated by regression analysis. As a is a coefficient that varies with site index, the equation can be rewritten to estimate dominant height as a function of site index.

$H = S * \exp\left(\beta(A^{-\kappa} - A_I^{-\kappa})\right)$	Eq. 4

Where, S is site index (m), and A_I is the index age at which site index corresponds to dominant height, in this case 20 years. With a site index per plot and site index invariant coefficients β and k, the equation was fitted to the data using non-linear regression.

1.3.4 Stand volume and basal area

To ensure consistent estimation of stand volume and basal area, one plot without basal area information was excluded from the analysis. The trees in this plot were too small to measure the dbh, and it would negatively affect the model estimation.

Total standing volume is directly estimated from stand characteristics using the following equation:

$$v = \alpha * A^{b_0 + b_1 S + b_2 R_S}$$ Eq. 5

Where, v is volume of the average tree in a stand (m³), A is stand age (years), S is site index (m), R_S is relative spacing (Burkhart & Tomé, 2012) and a and b₀ – b₂ are coefficients estimated with regression analysis. Multiplication of v with stocking (trees/ha) yields an estimate of total stand volume. There is a very close relation between volume, dominant height and basal area, which is estimated through the logarithmic equation:

$$G = b_0 V^{b_1} H^{b_2}$$ Eq. 6

Where, G is stand basal area (m²/ha), V is total stand volume (m³/ha) and H is stand dominant height (m).

1.3.5 Diameter distribution

As the forest stand is not homogeneous, that contains trees of various sizes, the distribution in tree size is key to accurately estimating tree volumes and size classes. For this the Weibull distribution was used to estimate the distribution of tree diameters in a stand (Bailey & Dell, 1973). The Weibull function has the cumulative form:

$$P(d) \; = \; 1 - exp(-((d-a)/b)^c$$ Eq. 7

Where, c is known as the shape parameter, b as the scale, and a is the origin. d is tree diameter, and $P(d)$ is the proportion of the stand less than or equal to d. The scale b is approximately the 63% point of the cumulative diameter distribution. The a parameter is manually estimated based on observed minimum diameter in the stand using the following equation:

$$a = b_0 + b_1 d_g$$ Eq. 8

Where, a is the origin parameter, d_g is the stand mean quadratic diameter (cm) and b_0, b_1 are the estimated coefficients. The shape parameter c is estimated using Mann's method (Mann, 1968), using the following equation:

$$c = \exp{(b_0 + b_1 d_g{}^\kappa)}$$ Eq. 9

Where, c is the shape parameter, d_g is the stand mean quadratic diameter (cm) and b_0, b_1 and k are the estimated coefficients. The scale parameter b is estimated using the method as described by Nord-Larsen & Cao (Nord-Larsen & Cao, 2006). This gives a b value that will ensure that the total basal area from the calculated Weibull distribution agrees with observed basal area. As at plot level there are too few trees to give good distribution characterisation and hence parameters estimates, data were aggregated based on mean quadratic diameter of the plots.

1.3.6 Thinning distribution

To estimate gross and merchantable harvesting volumes, the diameter distribution of the trees that were removed must be known. When a selective thinning from below is applied, as is normal practice in plantation management, smaller trees are preferentially removed. A function that simulates this was applied (Alder, 1979), which has the following form:

$$p_a = p_i{}^{1/L}$$ Eq. 10

Where, the p_a is a class of the diameter distribution after thinning for the same diameter point as the p_i class before thinning, and L is the leave fraction, or ratio of stocking after thinning to stocking before thinning.

$$L = \frac{N_{new}}{N}$$ Eq. 11

This method applies a greater thinning bias with heavy thinnings being more uniform in their effect. In order to estimate product volumes, the taper equation was used in a cross-cutting algorithm that used a set of product specifications.

1.3.7 Individual tree height

Since the taper equation uses total tree height as input parameter, the tree height per diameter class in the diameter distribution was estimated using the following equation (Gadow & Hui, 1998):

$$h = 1.3 + b_1 H^{b_2} d^{b_3 H^{b_4}}$$ Eq. 12

Where, h is total tree height (m), H is stand dominant height (m) and d is tree diameter at breast height (cm).

2   Results

2.1   Tree functions

Although the available section measurement data do not include trees older than 18 years, the estimated functions for total tree volume and tree taper showed a good statistical fit and little bias (Table 3). With new data becoming available, the functions can be further calibrated although it can be assumed that the model will also represent the later stages of rotation with sufficient accuracy.

The estimated function for individual tree height was found related to stand dominant height, which takes into account the relative size of the individual tree within a forest stand. While measurement of standing tree height is usually prone to measurement errors, the high number of measurements over the entire range of the rotation resulted in a good statistical fit of the individual tree height model (Table 3).

Table 3. Statistical values for the estimated functions for individual trees

Model		Coefficient	Coefficient value	R^2	RMSE
Eq. 1	Total tree volume	b_0	0.00004336	0.989	0.026
		b_1	1.7126		
		b_2	1.2045		
Eq. 2	Tree taper function	b_1	0.9911	0.962	1.479
		b_2	2.1608		
		b_3	0.7432		
		b_4	-0.5628		
Eq. 12	Tree height	b_1	0.05640	0.921	1.122
		b_2	1.6059		
		b_3	3.1787		
		b_4	-0.7978		

2.1.2 Stand functions

The Schumacher site index equation (Eq. 3, Eq. 4) was estimated with high precision (Table 4). Although the majority of the available data covered the first half of the rotation, the regular measurements from older stands with the same genetic material provided for sufficient data in the second half of the rotation. The model should regularly be updated in the coming years when new data become available, especially because the second half of the rotation is of most economic interest. Figure 1 shows the behaviour of the estimated model in comparison with average site index curves obtained from reference yield tables of West Africa (Dupuy, et al., 1999; Ganglo, 2020). The yield table from Ivory Coast showed a relatively fast growth at early stages of plantation development. However, the present study showed a slower development in early years and a more sustained growth in later stages of the rotation. This trend was similar to the yield table from Benin, where a

slower height growth was observed in the early stages of the rotation and sustained growth in later stages. Our data show that the dominant height growth patterns of teak in central Ghana lies in between the curves for Ivory Coast and Benin (Figure 1). The yield model from northern Ghana (Nunifu & Murchison, 1999) showed a significantly different growth pattern particularly for lower site indices with a rotation age of 20 years, which can be attributed to the different site conditions in this drier zone.
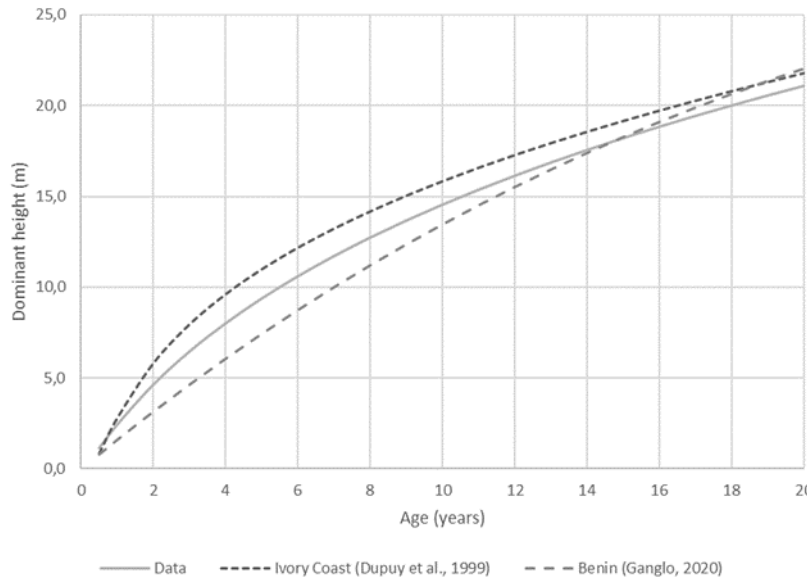

Figure 1. *Dominant height model for the teak plantations in central Ghana compared with other West African teak plantations in Ivory Coast (Dupuy, et al., 1999) and Benin (Ganglo, 2020)*

In order to cover the variability within the plantations, four anamorphic site index curves were defined each sepa-rated at the index age of 20 years by 3 metres height difference (Figure 2). This classification can be used for stand specific management based on the observed site class, while suffi-ciently covering the natural variation over time.
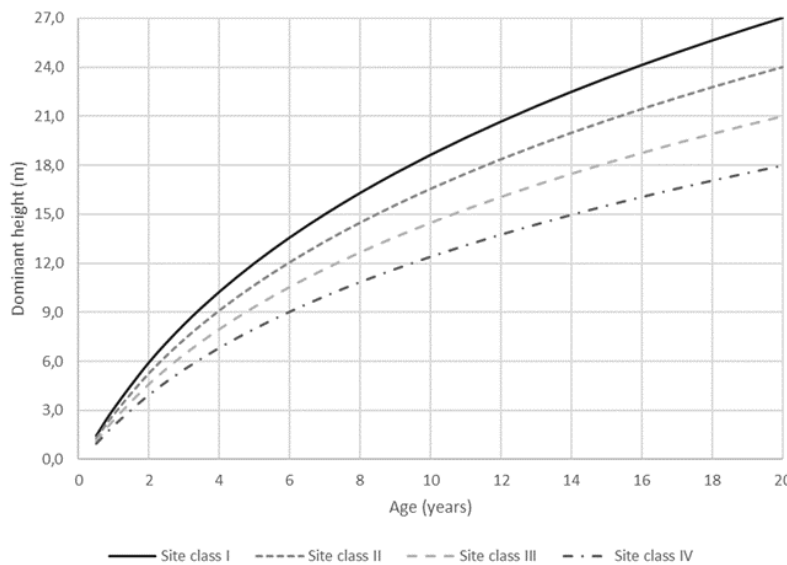

Figure *2. Site classes as obtained from the estimated site index function for teak in central Ghana*

In the estimation of the stand volume function various models were evaluated. Compared to other equations, the func-tion used in this study (Eq. 5) resulted in the lowest bias and a good statistical fit (Table 4). As the stand volume func-tion is a key component of the entire model, the reduction of bias is crucial from both a statistical as well as an op-erational perspective.

With stand basal area being closely related to stand volume and dominant height, the estimated stand basal area model (Eq. 6) yielded a good statistical fit (Table 4).

Table 4. Statistical values for the estimated functions for the total stand

Model		Coefficient	Coefficient value	R^2	RMSE
Eq. 3 & Eq. 4	Site Index model	b	-4.4301	0.952	0.837
		k	0.2218		
Eq. 5	Average tree volume	a	0.001038	0.958	0.024
		b_0	1.0265		
		b_1	0.04661		
		b_2	0.3416		
Eq. 6	Stand basal area	b_0	2.1298	0.995	0.332
		b_1	0.9682		
		b_2	-0.8775		

2.1.3 Diameter distribution

With the methodology used in this study, the yield model is expanded with a diameter distribution. This allows for estimation of product assortments that can be obtained from the forest stands. The obtained results (Table 5) yield a diameter distribution function that is consistent with the data over the entire rotation. Moreover, a methodology to estimate the scale parameter was used that yields estimates of the diameter distribution that are consistent with the observed basal area (Nord-Larsen & Cao, 2006). This will make the estimates of the product assortments estimated by using a cross-cutting algorithm more reliable.

Table 5. Estimated parameters for Weibull diameter distribution. The mean and standard deviations of goodness-of-fit statistic (Kolmogorov-Smirnov) for the fitted distribution are 0.149 (mean) and 0.084 (standard deviation)

Model		Coefficient	Coefficient value
Eq. 8	Weibull shape	b_0	-9.167
		b_1	0.7639
Eq. 9	Weibull shape parameter	b_0	1.493
		b_1	-3.482
		k	-1.338

3   Discussion

The full set of evaluated functions together form a comprehensive growth and yield model, which is the first published yield model for teak in West Africa that includes the possibility to predict product assortments based on diameter distribution and tree taper. The ability to predict wood flows and product assortments is crucial in developing a business case for teak plantations. This information forms the basis for potential investments in the establishment of these plantations, as it provides insight into the expected income over time.

The presented model was estimated using permanent sample plot data where the majority of the data origi-nates from relatively young stands, and has therefore a lower representation for the second half of the 20 year rotation. Care must be taken when extrapolating the model to age classes that are not included in the original measurement data. Despite these limitations, the model yields consistent estimates of the various equations with good statistical fit.

It should, however, be realized that trees show defects and losses, and that the entire length of a stem may not be merchantable. Often a minimum merchantable diameter is used up to which point the stem is utilizable. As teak may exhibit branching below this diameter, the merchantable length of the stem can be lower than the merchantable length based upon merchantable diameter alone. Besides that, teak trees often have defects along the merchantable section of the stem (pruning wounds, fluting). In order to enable for accurate product assortments and wood flows, a deduction has to be applied to take merchantable height, stem defects, and harvest losses into account. Realized harvesting data will provide the basis for this adjustment, as it will largely depend on the management history and planting material. Through good forest management and using im-proved planting material, it is anticipated that these losses can significantly be reduced over time.

The use of improved planting material that is more locally adapted is also expected to result in higher Site In-dex estimates. The current Site Index model is based on seed material from Ivory Coast and is in line with ref-erence figures from Ivory Coast and Benin. However, expected that further improvement can be made to the anticipated yields as a result of selecting genetic material which performs better under the specific local site conditions. Also, the history of degradation in the forest reserves is expected to play a limiting role in the site classification. As over time, soil organic matter is accumulating, this will result in higher site index estimates for the same teak genetic material during a second rotation.

4   Acknowledgements

5   References

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