# Yields of Eucalyptus and Caribbean Pine in Uganda

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This study develops yield models for two major plantation species in Uganda, *Pinus caribaea* and *Eucalyptus grandis*, and also provides recommendations for future growth and yield monitoring plots in plantations.

The yield models for both species are developed using similar techniques. The *Pinus caribaea* model is based on data from the Forestry Rehabilitation Project 1989-93 plantation inventory, which was converted from its archived format to an Excel file for analysis, and includes data from 868 *P. caribaea* plots. The *Eucalyptus grandis* data derived from 1990-96 permanent sample plots in periurban plantations established by the Forestry Department; and PSP data made available by Rwenzori Highlands Tea Company from their Eucalyptus forests. All data was summarised to the stand level to give age, dominant height, trees per ha, mean diameter, total volume, and volume to 10 cm top diameter. Volume equations were derived independently from earlier FAO studies in Uganda and international comparisons, together with a re-analysis of the FRP *Pinus caribaea* volume sample. Site index curves, volume-height-stocking, and self-thinning functions were derived by various graphical and statistical methods documented in the report. Finally, these various equations were assembled into two Excel-based spreadsheet models, one for each species.

The models are easy to use, and allow the forest planner to examine the effect of different site index, initial spacing, planting survival, and thinning regimes. Considering unthinned stands, the models show that *Pinus caribaea* has mean annual increments between 17 and 27 m<sup>3</sup>/ha/yr, depending on site index, with median values of around 22 m<sup>3</sup>/ha/yr. Optimum technical rotations (maximum volume production) vary from 22 to 33 years on the same site range. *Eucalyptus grandis* has an MAI range from about 20-47 m<sup>3</sup>/ha/yr, with optimum rotations of 7-13 years. Production on both species is very sensitive to management practice. Failure to follow best silvicultural practice results in major losses in yield.

The models have some limitations, and could be improved in future with an investment of further time. The Eucalyptus model does not cover coppice stands. Neither model is fully sensitive to the effect of spacing or thinning on diameter growth, and will underestimate the diameter response to thinning.

The monitoring recommendations for future PSPs are to use circular plots of 12 m radius (452 m2). All trees should be measured for diameter, scored for form and qualitatively assessed. The four largest dbh should be measured for height. Trees should be numbered with tags, with multiple stems and coppice using a sub-numbering scheme. Point of measurement (1.3 m) should be painted on each tree. The proposed NFA should have a specialist unit for PSPs and inventory and centralise these operations using core staff to ensure good quality standards in field work. Immediate priority should be given to establishing PSPs in identifiable Research Plots for which records exist, and protecting them against further losses. Re-measurement of the Periurban PSPs which still exist is highly desirable. New plantations should have PSPs established in the second year after planting at a rate of 1 per 5 ha. This should however not be a statutory requirement for the private sector, and should apply primarily to NFA's own operations, with linkage to a system for continuous forest inventory and management information reporting.

The *Pinus caribaea* and *Eucalyptus grandis* yield models, data summaries, this report, and some related presentations can be downloaded from <a href="http://www.denisalder.com/uganda03">http://www.denisalder.com/uganda03</a>.

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#### List of abbreviations and acronyms

BA	Basal area
BAF	Basal area factor
CAI	Current Annual Increment (m³/ha/yr)
CFI	Continuous Forest Inventory
dbh	Diameter at breast height (1.3m)
DOS	Microsoft Disk Operating System (MS-DOS)
EC	European Community
FRMCP	EC-Uganda Forest Resources Management and Conservation
	Programme
FRP	Forestry Rehabilitation Project
G&Y	Growth and yield
GIS	Geographical Information System
GPS	Global Positioning System
IDA	International Development Association (World Bank)
MAI	Mean Annual Volume Increment (m³/ha/yr)
MIS	Management information system
NBS	National Biomass Study
NFA	National Forest Authority
PPP	Periurban Plantation Project
PSP	Permanent Sample Plot
RHTC	Rwenzori Highlands Tea Company
SI	Site Index
UFA	Uganda Forestry Association
UFD	Uganda Forestry Department

#### Software trademarks

Some software packages in very widespread use have now entered into daily usage as part of the language. However, we acknowledge that Excel, Access, and Word, where used as proper nouns, and FoxPro and Visual Basic are trademarks of *Microsoft Corporation*. Turbo Basic and Dbase are trademarks of *Inprise Corporation* (formerly Borland). ArcView is a trademark of *ESRI Corporation*.

# Algebraic and forestry symbols

The following list gives the standard algebraic symbols used in the text. As far as possible we have followed the recommended standards of IUFRO. Units used are shown in brackets.

٨	an asymptotic coefficient used in various equations
A	Tree diameter at breast bright (1.2 m) in sm
a	The diameter at breast height (1.5 m) in cm.
<i>a</i>	The differential operator.
D <sub>g</sub>	Stand mean basal area diameter (cm).
<i>e</i>	The mathematical constant 2.71828
F	A cumulative frequency
f	form factor
f(x)	Any function of x
<i>f</i> ′( <i>x</i> )	The derivative of the function with respect to x
G	Stand basal area (m²/ha)
g	Tree basal area (m²)
h	Individual tree height (m)
H <sub>10</sub>	Dominant height at a specified age, eg. 10 years.
H <sub>d</sub>	Stand dominant height (m)
k	a shape coefficient, used in various equations
m	a scale coefficient, used in various equations
n	Number or count of items, eg. number of trees on a plot.
N	Stocking, or trees per ha.
P	A probability or proportion
p	plot size in ha.
ן מ	the constant 0.00007854, or $\pi/40000$ .
r	rate coefficient, in some exponential equations
R	Ratio, usually for merchantable volume conversion factors
S	Site index generally $H_4$ at a specified base age
t	Stand age in years
V	Stand total volume overbark $(m^3/ha)$
v	volume of a tree $(m^3)$
V	Stand volume to a 10 cm ton diameter overbark $(m^3/h^3)$
v 10	intercent coefficient in linear equation forms [alpha]
0	along a selficients in linear equation forms [hete]
β	slope coefficients in linear equation forms [beta]
0	a top diameter limit [delta]
$\Delta V$	change in volume, volume increment [delta V]
π	the mathematical constant 3.14159
Σ	Summation operator [sigma]
ζ	stand density index (zeta)

Caribbean Pine (*Pinus caribaea*) and *Eucalyptus grandis* are two of the most widely planted exotic plantation species in Uganda. Both have proved themselves to be very successful, silviculturally robust, and adaptable within a range of sites. However, apart from some early studies by Kriek (1969) and Kingston (1972a, 1972b, 1972c), there is little published information on their growth and yield in Uganda.

The present study has been commissioned under the EC-supported *Uganda Forest Resources Management and Conservation Programme* to help remedy this deficiency. The author's terms of reference were to compile and analyse available growth and yield data, provide practical yield models, and to recommend monitoring methods to acquire further and better quality yield data in future.

In undertaking this study as a relatively short consultancy assignment, the authors were fortunate that there exist some good data sets for the two species. These include the forest





inventory of softwood plantation undertaken under IDA auspices in 1989-93, the permanent sample plots (PSPs) of the National Biomass Study which have been established in Periurban plantations, and the PSPs established by Rwenzori Highlands Tea Company (RHTC) in their estates. In addition there is a great deal of international literature on the growth and yield of the two species, much of which was directly relevant and helpful.

The map shows the main locations involved. *Pinus caribaea* is mainly planted at Nakwaya, Katugo, Kikonda, the Mwenge group of reserves near Fort Portal, and at Lendu, to the north of Lake Albert. The *Eucalyptus* plantations of RHTC are also in the west, near to Fort Portal. The Periurban *Eucalyptus* plantations were near Kampala at Namanve, north of Jinja at Nsube/Mutai, in the east at Tororo, in the south-west at Kyahi, and near Katugo at Mbale.

This report describes the analysis of this data, and the development of yield models based on it. It is divided into two main sections, one for each of the two species. We also suggest standards for future growth and yield monitoring, especially within the proposed National Forest Authority (NFA).

# Sources of data

The data used for developing the yield functions for Caribbean Pine derive from the 1989-93 plantation inventory sponsored by the World Bank Uganda Forest Rehabilitation Project. Two of the authors (Alder and Elungat) had been involved in biometric and

computational aspects of that project, and fortunately had archived copies of the original data files and programs. The data comprised:

- **□** Tree volume measurements from destructive sampling for *Pinus caribaea* for 594 trees.
- □ Temporary sample plot measurements for 2859 plots, including 868 plots in *Pinus caribaea* stands.

Programs were written in Turbo Basic for DOS<sup>\*</sup> to convert both data sets from their original binary formats into comma-separated text files which could be imported into Excel for further processing.

The volume measurements comprised both the raw measurements and a summary file which included over and underbark volumes to 5, 10, 15 and 20 cm top diameters (over or under bark), tree dbh, total height and stand dominant height at the point where the sample tree was taken.

The sample plots were based on point sample plots, which used variable basal area factors (usually 2 or  $4 \text{ m}^2/\text{ha}$ ). On each plot, the counted trees were measured for dbh, stem quality was recorded using various codes, and dominant height estimated by sampling the two dominant trees (largest diameter) at each point. For the purposes of this study, only Caribbean Pine data was used, but the entire data set has been converted to Excel and is available for further work on other species.

								Plan	ting y	ear									Year of
Forest	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	Total	inventory
Namafuma						12		27	17	26	16							98	89
Lendu	8	2	13	3														26	93
Okavu Reru					4	8												12	93
Katugo						7	8	9	26	12	18	59	60	38	27	26		290	90
Nakwaya											7	62	52					121	90
Lukuga			4		1	9			15	12			20					61	90
Zimwa						12		19		15			6					52	90
Kikonda													61	44			2	107	90
Oruha					4	7	8	3										22	92
Kyehara					2				8									10	92
Kikumiro					1	13	1	1	4	2								22	92
Kanyawara			2	1	5	12	1	16	6				1					44	92
Total	8	2	19	4	17	80	18	75	76	67	41	121	200	82	27	26	2	865	

# Table 1 : Distribution of Caribbean Pine plots in the UFRP 1989-93 Plantation Inventory by forest and planting year

<sup>\*</sup> This was the computer language originally used for these data in 1989-90, and it was easier to write the conversion routines using it, rather than a modern language such as MS Visual Basic.

# Tree and stand volume equations

Volume equations for *Pinus caribaea* were developed under the UFRP and have been used by the UFD since that time (Alder, 1990). However, it has been found that when extrapolated outside the height-diameter limits of the original sample, the volumes for higher top

diameters in some cases exceed those for lower diameters, in a way that is clearly impossible. It was therefore necessary to recalculate the equations using a different technique, so that this could not happen.

#### Sample tree size distribution

Table 2 shows the sample distribution of the felled *Pinus caribaea* volume trees. When applying the models, it should be appreciated that height-diameter combinations which lie outside the sample range may result in biased volume estimates.

# Table 2 : Diameter and height distribution of Pinus caribaea felled for volume sampling during the 1990 UFRP study

Dbh			Height	class, m			
class, cm	5-10	10-15	15-20	20-25	25-30	30-35	TOTAL
10-15		2					2
15-20	1	6	1				8
20-25		3	12	3			18
25-30			9	14	1		24
30-35			6	17	1		24
35-40			5	21	11		37
40-45			4	29	17		50
45-50			1	17	12	2	32
50-55				5	1	1	7
55-60					1		1
TOTAL	1	11	38	106	44	3	203

# Individual tree volume form factors

For individual tree volumes, a simple form factor model was used, expressed by the equation:

{1}

 $v = f.q.d^2h$ 

where *v* is tree volume, *d* is tree diameter, and *h* is tree total height. *q* is the constant 0.00007854 (or  $\pi/40000$ ), *f* is the form factor, or ratio of tree volume to the volume of a cylinder of the same diameter and height.

The mean form factor for overbark volume to a 5 cm top diameter is 0.420 with an  $R^2$  of 97.3% (Figure 2). The underbark form factor to a 5 cm top diameter underbark is 0.328 with an  $R^2$  of 96.2% (Figure 3). The lines shown on the graphs are linear regressions with a forced intercept of zero, whose slope therefore corresponds to the form factor. The points represent individual sample trees.

Individual tree volume equations based on tree height and diameter are not very useful in yield models as it is not usual to have information about individual tree heights. Even in inventories to estimate standing volume, it is not desirable to measure heights individually, and usually only stand dominant height is available. These tree form factors are therefore presented here only for completeness.



Fig 2: Pinus caribaea - Overbark volume form factor to 5 cm top





It should be noted in the above graphs that the cylindrical volume is always calculated using normal tree dbh overbark and total height measure, in cm and m respectively.

# Volume equations based on stand dominant height

Volume equations based on stand dominant height are more useful for most purposes than those based on individual tree height. Volume equations are mostly used to calculate the volumes of stands or plots. In this case, individual tree heights are not usually measured and must be calculated indirectly from a locally constructed diameter-height regression if a volume equation based individual tree height is to be used. This introduces bias and additional uncertainty into the volume estimation process. It is more accurate and precise\* to use a volume equation that is directly based on stand dominant height when estimating stand volumes.

Dominant height, as noted and discussed further in the section on Site Index, is defined as the mean height of the 100 largest diameter trees per ha.

The stand volume equations for over and underbark volumes to a 5 cm top diameter fitted by regression analysis were, respectively:

$V_o = 0.00004534 \ d^{1.8875} \ .H_d^{1.0304}$	{2}
$V_u = 0.00001638 \ d^{1.9497}.H_d^{1.2006}$	{3}

The coefficients of these equations were both estimated by multiple regression, using the linear form:

$$\ln(V) = \alpha + \beta_1 . \ln(d) + \beta_2 . \ln(H)$$
<sup>{4}</sup>

In the linear form {4} the R<sup>2</sup> for the overbark and underbark equations were 99.3% and 98.8%, respectively. There were 202 sample trees. The error distribution from equation {2} is shown in terms of percentage errors in the histogram in Figure 4. For the overbark equation, the mean error is 0.2% of predicted volume, and 95% of the sample had errors between –14% and +12%. For the underbark equation, mean error was 0.5%, and 95% of the sample had errors between –20% and +18%. Note that percentage error, as defined here, is calculated by:

Error % = 
$$(V - V_p) / V_p \times 100$$
 {5}

where V<sub>p</sub> is the predicted volume from the equation, and V is the actual measured volume.

### Fig 4 : Error distributions for the overbark volume equation for Pinus caribaea



<sup>\*</sup> In biometrics, precision refers to the random error associated with an estimate, and accuracy to the bias.

#### **Conversion factors for merchantable limits**

The above equations refer to volume to a 5 cm top diameter. This is very close to what is normally called total volume, or stem volume to a theoretical branch size of zero calculated by a conical or parabolic extrapolation. For practical use, volumes are required to larger top diameter limits that reflect normal timber utilization for fuelwood, timber, chipwood, etc. The UFRP volume equations achieved this by fitting separate volume equations to limits of 10, 15 and 20 cm top diameter, both over and underbark, giving 8 equations in all, including the 5 cm ones. Some of these equations cross over, giving contradictory results (*eg.*. 20 cm top diameter volumes greater than those to 15 cm top diameters for some height-diameter combinations).

To achieve results that would be more parsimonious in terms numbers of coefficients, and also avoid any inconsistencies if the equations were extrapolated, a method proposed by Clutter *et al* (1983) was adopted. This has been quite widely used, for example by Shiver & Brister (1993) for *Eucalyptus saligna* in Kenya and Alder (1998) for *Pinus radiata* in Ecuador, and performs well. The equation has the form:

$$\mathbf{v}_{\delta} = \mathbf{v}(1 - \mathbf{a}.\delta^{\mathbf{b}}.\mathbf{d}^{\mathbf{c}}) \tag{6}$$

Here  $v_{\delta}$  is the volume to a top diameter limit  $\delta$ , v is tree total volume, and d is tree diameter. This can be fitted by regression using the linear form:

$$1 - v_{\delta}/v = \alpha + b \cdot \ln(\delta) + c \cdot \ln(d)$$
<sup>{7}</sup>

This model allows a conversion ratio  $(v_{\delta} / v)$  to be calculated for any top diameter limit. For *Pinus caribaea*, the calculated coefficients for overbark conversion were:

$$v_{\delta} = v(1 - 0.5747.\delta^{-3.3624}.d^{-3.2654})$$
 {8}

This equation had an R<sup>2</sup> in its linear form {7} of 96.8%, with 590 data points (202 trees and 3



Fig 5 : Predicted and actual merchantable volume conversion ratios for *Pinus caribaea* The predicted values are shown as lines; the actual values are points.

top diameters). The underbark conversion equation had the coefficients shown below, and an  $R^2$  of 96.5%.

$$v_{u\delta} = v(1 - 0.6565.\delta^{3.3972}.d^{-3.3275})$$
<sup>{9</sup>

The equations were tested by plotting the actual volume ratios  $(1 - v_{\delta}/v)$  against the tree volumes for the three different top diameter limits (10, 15, and 20 cm), giving the results shown in Figure 5 for the overbark equation {8}. Results for the underbark equation are similar. It can be seen that the predictive power of the equation, over a range of tree sizes and diameter limits, is excellent. Note that in Figure 5, both axes are plotted using logarithmic scales.

# Application of the tree volume equations

The key results in the foregoing analysis are the total volume equations {2} and {3} for over and underbark volume to a 5 cm top diameter, and the conversion ratio equations {8} and {9}. These were used to calculate plot volumes based on individual tree diameters and plot dominant heights. The plot volumes were then used in the further analysis of total yields.

#### Site Index

Site index curves relate height growth to age, and are of fundamental importance in forest yield models. Other factors of yield, such as volume or mean diameter can be more accurately predicted from height and age than

from age alone. Height at an early age of stand development is also a strong indicator of the subsequent performance of a stand.

Stand height is most commonly measured as stand dominant height. The most widely accepted definition of dominant height is the mean height of the 100 largest diameter trees per ha, excluding those of abnormal height development (broken tops, diseased trees, double stems etc.). Dominant height has several advantages over simple stand mean height. It is independent of stand density, and unaffected by thinning from below (the normal practice in plantation thinning). It is not confused by coppice re-growth or natural regeneration, which renders mean height very difficult to define in less uniform plantations. Its sampling on a small plot is simpler to define than mean height.

#### Review of site index curves for Pinus caribaea

There are several studies which give site index curves for *Pinus caribaea*, including a provisional one by Kingston (1972b) for Uganda for young stands up to 12 years old. These include Fierros-Gonzales *et al* (1992) for Oaxaca, Mexico; Scolforo (1992) for Sao Paolo state, Brazil; Adegbehin & Onyibe (2001) for northern Nigeria; and Liegel (1991) for several countries of the Caribbean Basin. The guide curve for Kingston's Uganda study is shown in Figure 6, together with curves from Mexico, Brazil, and Nigeria for equivalent site index values, to show their relative shapes. Liegel's study did not include equations or tabulated values from the site index curves, so they have not been included in the comparisons. His paper does however include the stand summary data from sample plots in Costa Rica, Jamaica, Trinidad, and Venezuela which could be useful for comparative work in some contexts; in the present case, time did not permit the possibilities to be fully explored.

Kingston's 1972 curves were based on the empirical equation:

$$\ln(H_d) = -0.2906 + 1.6973 \ln(t) - 0.09323 t$$
<sup>(10)</sup>

where t is stand age in years and  $H_d$  is dominant height. His original equation based on imperial units (feet) has here been recalibrated to give a result in metres. This is not an asymptotic function (ie. one that tends to a maximum height as age increases), but rather has



Fig 6 : Comparison of Caribbean Pine height growth curves from Uganda, Brazil, Mexico and Nigeria

a maximum at a certain age, after which it decreases. Figure 6 shows Kingston's median site index curve with the extrapolation as a

index curve with the extrapolation as a dotted line. Height declines after about age 18, making this curve form unsuitable as a model for longer rotations.

Most modern site index curve studies tend to be based on sigmoid functions, including the Schumacher, Chapman-Richards and Gompertz models as common examples. Sigmoid curves have two key features, illustrated in Figure 7. These are an asymptote, or maximum limit towards which the y value tends as the time axis goes to infinity; and an inflection point at which the slope of the curve is a maximum. The various kinds of models have different relative relationships between the inflection and asymptote that determines their suitability for different sets of data.

Fig 7 : Parts of a typical sigmoid curve



The Mexican curve (Fierros-Gonzalez et al, 1992) is based on the Schumacher equation:

$$H_d = A.exp(-m.t^{-k})$$
<sup>{11</sup>}

This has been widely used for site index curves in forestry, and special methods have been developed for fitting the parameters to permanent sample plot data (Bailey & Clutter, 1974; Clutter et al., 1983; Alder, 1980). However, its shape is not always suitable, as appears to be

the case here, where later height growth is too linear, and early growth, at the foot of the curve, may be underestimated.

The Nigerian work of Adegbehin & Onyibe (2001) is based on the Gompertz function, which has been popularised there through the work of Nokoe (1978). This has the form:

 $H_d = A.exp[-exp\{-m(t-k)\}]$  {12}

where A, m and k are parameters,  $H_d$  and t are as previously defined. The Gompertz function is somewhat constrained in its shape as the inflection point must occur at  $H_d = A/e$ . Depending on how the data constrains the curve, this may either force the foot of the curve to be too high, as seems to be the case in Figure 6, or force the asymptote to be too flat. However, in some cases the Gompertz model may be perfectly suitable.

Scolforo's 1992 site index curves are based on the Chapman-Richards function, also known as the von Bertalanffy-Richards equation. In its 'Bertalanffy' form this is written as:

 $H_{d} = A.[1 - \exp(-k.t)]^{1/(1-m)}$ [13]

The 'Chapman' form is the same except that the 1/(1-m) term is simply expressed as c. Because the m coefficient has a simpler relationship to the inflection point than the c parameter, we will prefer the former, as given in equation {13}. For a thorough (and mathematical) discussion of all these sigmoid models, their antecedents and relationships, see Zeide (1993).

Depending on the data, the Chapman-Richards function will generally represent early height growth better than the Schumacher equation, which tends to be too flat near the origin; however, the Chapman-Richards may on the other hand flatten off to an asymptote too early. It will be found that different species are best represented by one or other of the curves, and that the choice may also be influenced by the age distribution of the data available. In Figure 6, it can be seen that different workers have selected different models for the same species, *Pinus caribaea*. Both Scolforo (1992) and Fierros-Gonzalez *et al*, (1992) carefully examined the alternatives, and arrived at different choices (Chapman-Richards versus Schumacher).

# Method of fitting curves for Uganda

For the new Uganda site index curves for *Pinus caribaea*, the temporary sample plot data from the UFRP inventory were displayed graphically, as shown in Figure 8. The youngest stands were about 14 years old. To provide information about the early growth pattern, Kingston's (1972) median curve was overlaid on the graph, together with his upper and lower curves representing plus or minus one standard deviation from the logarithmic regression. It was assumed that Kingston had based his work on samples from the same plantations as those which occurred in our data set aged 20 and above (1970 plantings and earlier), and his median line, being a regression curve, accurately represented the growth trend of the young stands.

A test bed was then constructed in Excel which would allow various sets of site index curves, based on the Schumacher, Gompertz or Chapman-Richards models, to be manipulated using the mouse to achieve a fit by eye that coincided with Kingston's curve and the current data set as well as possible. This is done by defining two control points on the graph, which can be moved with the mouse to change their values. One control point defines the asymptote A, and the other the height and age of the inflection point. From the latter, the k and m coefficients can be computed.

It was found by this process that the Gompertz function was unsuitable, due to its inherent constraint as noted above of having a asymptote that has a fixed relation to the inflection point. The Schumacher model fitted older stand growth well, but did not model height growth in the first three years in a credible way. The Chapman-Richards function was found generally most suitable.

However, it was also found that the median, upper and lower guide curves for the site index system had to be modelled independently to give a good representation to the data. It was not possible to produce a simple set of proportional (polymorphic) or parallel (anamorphic) curves.

Having obtained the best set of curves by this visual adjustment process, as depicted in Figure 8, the coefficients were examined graphically against the site index of each line, and a simple function selected from Excel's repertoire of regression models to 'harmonize' the curves. This provides a mean of estimating each coefficient from site index, and allows intermediate curves to be drawn.



Fig 8 : Appearance of the Excel graphical tool for locating site index curves interactively

### Final site index model for Pinus caribaea

The final mathematical model for the site index curves was:

 $H_d = A.[1 - exp(-k.t)]^{1/(1-m)}$ 

where:

$$m = -0.01784 H_{10} + 0.4847$$

k = 0.0606

 $A = H_{10} / [1 - exp(-k.10)]^{1/(1-m)}$ 

{14}

These are drawn as a set of curves in Figure 9. For consistency with Kingston and to allow comparison with the Eucalyptus curves, a base age of 10 years is used.  $H_{10}$  is dominant height at 10 years, and is also called site index and symbolised as S.



**Fig 9 : Site index curves for Pinus caribaea var. hondurensis in Uganda** Points are temporary sample plots from 1989-93 FRP inventory. Site index base age is 10 years.

#### Estimating site index

If height and age are known, site index can be estimated graphically from Figure 9. Equation  $\{14\}$  can also be solved for site index by re-writing it in the form x=f(x). Then from standard numerical analysis theory, if  $x_0$  is an initial estimate of x, the system can be solved by the iteration:

$$x_{i+1} = f(x_i)$$
 {15}

until the difference  $|x_{i+1} - x_i|$  becomes negligible<sup>\*</sup> (Stark, 1970, p 69-85). For the site index curves, the best initial guess is the median site index of about 14. Equation {14} can be rewritten in the style of {15} to solve for site index S in this way, using the form shown below. In this, S<sub>i</sub> is the estimate of site index from a previous iteration (starting at S<sub>0</sub>=14), t and H<sub>d</sub> are the age and dominant height of the stand for which site index is to be estimated.

$$S_{i+1} = H_d \left\{ \frac{1 - \exp(-0.606)}{1 - \exp(-0.0606.t)} \right\}^{\frac{1}{0.01784S_i + 0.5153}}$$
<sup>[16]</sup>

The following Visual Basic macro implements this algorithm and can be used as a worksheet function within Excel to return site index given height and age. To use the macro, it should be cut and pasted into a module attached to the worksheet. Modules can be edited via the Visual Basic editor, accessible from the Excel *Tools/Macro/Visual Basic editor* option.

This can be used in Excel by inserting the function =GETSI(H,T) into a worksheet, where H is any cell containing dominant height, and T any cell containing the corresponding stand age. If the function returns -1 or an error value, then bad height/age values have been given. The values should be reasonable ones for Caribbean Pine stands.

# Factors influencing site index

Using the above algorithm, site index values were calculated for each of the *Pinus caribaea* inventory plots. These were summarised by forests with mean, standard deviations and 95% confidence limits, as shown in Table 3. The forests are sorted from lowest to highest mean site index. The rainfall and altitude information from old management plans for each forest are also shown. Figure 10 shows the mean site index values and confidence intervals. Forests with non-overlapping confidence limits will have statistically significant differences in their mean site index values.

<sup>\*</sup> Note that not all functions are guaranteed to converge with this rule. However the site index model has been tested with all the *P. caribaea* data and the method appears to behave robustly.



It can be seen from Table 3 that there is a clear association of site index with altitude, but the linkage to rainfall is less obvious. It should be borne in mind that these rainfall statistics, taken from old working plans, may not be reliable.

Generally, the higher altitude forests around 1500 m show a much better performance for Caribbean Pine than those at lower altitude. This could also be Table 3 :Caribbean Pine site index statistics by forest reserve Forests are ordered from worst to best site index

Forest		Altitude	Rainfall	Plots	Site	index
Code	Name	т	mm		Mean	Std.Dev.
38	Nakwaya	1290	1219	121	10.4	1.6
40	Zimwa	1327	1143	52	10.5	2.0
11	Namafuma	1250	1125	98	11.8	1.8
39	Lukuga	1290	1219	61	11.9	2.1
29	Katugo	1070	1125	290	12.0	1.7
41	Kikonda	1110	1062	107	12.7	1.9
51	Kyehara	1500	1400	10	14.1	1.7
52	Kikumiro	1500	1400	22	14.2	1.9
50	Oruha	1500	1400	22	14.7	2.4
25	Lendu	1520	1250	26	15.4	2.8
54	Kanyawara	1500	1400	44	15.5	2.5
26	Okavu Reru	1500	1125	12	16.0	1.5

associated with soil factors, and rainfall seasonality.

# The stand growth function

A growth increment function that predicts increment over a year for a stand from its age, stocking and site index is the core of a growth and yield model. Stand growth may be expressed in terms of diameter

increment, basal area increment, or volume growth. Models based on diameter increment are typically those with a more detailed internal structure, such as Alder's (1979) model for *Pinus radiata, Cupressus lusitanica* and *Pinus patula* in East Africa. In South Africa, modelling growth via basal area increment has been a common technique for several species, as exemplified through the work of Pienaar & Turnbull (1973). Several such models are cited in the section on *Eucalyptus grandis* (see page 37), and have been developed for sub-tropical pines such as *P. patula* and *P. radiata*. This has mainly been due to the dominant influence of the C.C.T. series of experiments in South Africa, and of Marsh's Hypothesis, which is essentially a growth and thinning response model framed in terms of basal area (Burgers, 1971; Marsh & Burgers, 1973).

Stand growth models based directly on the prediction of volume are also common and there are many examples in the literature for various species. For Caribbean Pine, Scolforo (1992) and Scolforo & Machado (1996) have developed a model for thinned stands in Sao Paolo State, Brazil that use various forms of yield function to predict total volume from age, site index and basal area. Green et al (1992) use Bayesian estimation to fit the equation:

$$\ln(V) = \alpha + \beta_1 t^{-1} + \beta_2 \ln(H_d) + \beta_3 \ln(N)$$
<sup>[17]</sup>

which predicts volume (V) from age (t), dominant height (H<sub>d</sub>) and stocking (N). They apply this to country data from Jamaica, Puerto Rica, Costa Rica, Trinidad and Venezuela which is published in plot summary form in Liegel (1991) to fit models by soil and site groups. Liegel (1991) himself uses the same equation, but fitted by conventional regression, to derive country-average models, but unfortunately only presents results in tabular form, and not the original equations. His listing of plot data for all the main stand parameters from these countries is however extremely useful for testing and comparison of models from other regions, including Uganda. In Nigeria also, a model has been developed for *Pinus caribaea* based on the Gompertz function (Nokoe, 1978) that predicts standing volume from site index and age for average stockings.

# The logical basis of the yield model

For the present work, an approach similar to that adopted by Alder (1996) for *Pinus radiata*, and Alder & Montenegro (1999) for *Cordia alliodora* in Ecuador has been followed. This involves fitting a function with the general form:

 $V = f(H_d, N)$  {18}

This is based logically upon Eichorn's Rule, first noted in 1904 by Eichorn (as described in Assmann, 1970, p. 161) that the standing volume of a species at a specified stand height is independent of site index. Eichorn also noted that the V/H relationship is logarithmic in form (ie. a straight line on log-log graph paper), and tends towards a limit that is independent of initial stocking at higher stand densities.

Ideally, one would fit function {18} in some empirical form to data from a spacing trial, representing stands that have grown at constant stocking from planting until the age used in the model. The model can then be differentiated to directly give current annual increment,  $\Delta V$ .

In practice with temporary sample plot data, as with site index curves, it is difficult to know the growth history of individual plots. It is possible, as many authors do, to work with regressions that treat current stocking as if it had always been constant. This results in models that tend to underestimate somewhat the responsiveness of species to increases in spacing. Alternatively, one can artificially adjust a fitted regression model to reflect the degree of response that has been observed for the species in experiments from other locations. In the present case, the authors opted for the former methodology, although the latter approach could be introduced if more time were to become available in future for review and refinement of the model. Using direct regressions is a conservative approach that will underestimate thinning responses in terms of volume increment and diameter growth.

#### The empirical model for Uganda

The general model described by equation {18} was fitted in two stages in order to explore exactly the sensitivity of the model to stand density. The first step, shown in Figure 11, was a regression relating dominant height to mean tree total volume.



Fig 11 : Volume – dominant height regression for Pinus caribaea Volume is to 5 cm top overbark. The regression is  $ln(V/N) = \alpha + \beta$ .  $ln(H_d)$ 

This fitted regression model had the form:

$$\ln(V/N) = -7.2328 + 2.1619 \ln(H_d)$$
<sup>[19]</sup>

The dependent (y) variable was the  $\log_e$  of the mean tree volume, and the independent (x) variable is the  $\log_e$  of dominant height. There were 865 plots in the regression, and R<sup>2</sup> was 80.0%.

Next, the ratios of actual to predicted mean tree volumes were plotted against mean spacing, giving the data and model shown in Figure 12. The regression equation in this case was:

$$V/V^* = 0.5046 Ln(\sqrt{10000/N}) + 0.2638$$
 {20}

This fitted the data with an  $R^2$  of 40.2%. This relation implies that about 40% of the variation shown on Figure 11 about the regression line could be explained in terms of differences in current stocking. In equation {20} V is the actual total volume, and V\* is the predicted volume derived from a transformation of equation {19}. These two equations can then be combined into a single formula to predict standing volume from dominant height, as:

$$V = (0.5046Ln(\sqrt{10000/N}) + 0.2638).exp[-7.2328 + 2.1619 ln(H_d)+ln(N)] \{21\}$$



Fig 12 : Model for the residual effect of spacing on mean tree volume

Equation {21} was tested as a model for directly estimating standing volume by plotting its predicted values against observed values of volume, as shown Figure 13. This shows that equation {21} has an R<sup>2</sup> of 90.9%, leaving about 9% of the variation in stand volume unexplained in terms of current stocking and dominant height.



Fig 13 : Actual versus predicted stand volumes from the yield model equation {21}

#### **Current annual increment**

A dynamic model that can encompass different thinning regimes needs to be based on current annual volume increment, rather than accrued volume yield in a static model such as equation {21}. For this purpose we may note that  $H_d$  in eq. {21} is a function of time, and the model is differentiable in respect of time to directly give CAI. However, this is quite cumbersome mathematically, especially as N itself may also be time-dependent when self-thinning is occurring. It was found that the best way to predict CAI was through a finite difference model which appears as follows:

$$\Delta V = f_v(H_t, N_t) - V_{t-1}$$
<sup>(22)</sup>

Here  $\Delta V$  is CAI between time *t*-1 and *t*,  $f_v(H_t, N_t)$  is the static volume calculated from equation {21} using current dominant height and stocking and V<sub>t-1</sub> is the standing volume at the previous time period *after* deducting any thinning removals in that period. The standing volume V<sub>t</sub> is itself the sum of the  $\Delta V$  values for each time interval, less thinning removals.

#### Stand underbark volume to 10 cm top

The above models predict overbark volumes to a 5 cm top. The yield model also requires underbark volume to a 10 cm top diameter. This was derived from overbark volume by fitting a logarithmic regression model to the FRP inventory plot summaries with the underbark 10 cm stand volume as the dependent variable and mean basal area diameter and overbark 5 cm stand volume as the independent variables. This gave the following fitted equation :

$$V_{10ub} = 0.23232 D_g^{0.30142} V^{1.02238}$$
 {23}

Here,  $V_{10ub}$  is the stand underbark volume to a 10 cm top, in m<sup>3</sup>/ha;  $D_g$  is the stand mean basal area diameter, in cm; and V is the stand overbark volume to a 5 cm top, in m<sup>3</sup>/ha. The R<sup>2</sup> of this regression was 99.86% with data from 867 plots.

#### Mean diameter estimated from total volume

It is also necessary in the yield model to be able to predict mean diameter from total volume and stem numbers. This was done via the fitted regression:

$$D_g = 42.17 * V^{0.3107} N^{-0.3425}$$
 {24}

This regression, fitted to the data from the 867 sample plots, had an R<sup>2</sup> of 93.6%.

#### Self-thinning

Planted stands of trees will undergo self-thinning in a predictable way as canopy closes and the less advantaged trees are suppressed by the faster growing ones. There is quite an extensive literature on this

topic (eg. Weller, 1987; Zeide, 1987; Bredenkamp & Burkhart, 1990; Zeide 1991). Reineke (1933) noted that when trees numbers are plotted against quadratic mean diameter on log-log paper, a straight line occurs for fully-stocked stands which has a slope of approximately -1.6 (the Reineke Line, as it is now called). The so-called -3/2 Power Law in ecology (see citations above) describes the relationship on a similar basis between stem numbers and individual trees biomass or volume, although the coefficient of the log-log slope is, on closer examination, species and management sensitive, and only approximates the standard values proposed by some authors. However, there is no doubt that there is a limiting self-thinning process that appears linear on logarithmic axes, and this provides a useful basis for modelling.

#### Estimating the self-thinning line in the FRP inventory data

Figure 14 shows tree numbers plotted against dominant height on log-log scales. A limiting value for stand density represented by a slope of -1.6 is assumed, and shown as the solid line.

10000

Several parallel stand density lines are drawn, marked as stand densities of 25%, 50% and 75% of the limiting line.

The lines are represented by an equation in the form:

N= 232093 
$$H_{d}^{-1.6055}$$
. $\zeta$  {25}

where N is stocking in trees/ha, Hd is dominant height in metres, and  $\zeta$  (zeta) is a stand density coefficient between 0 and 1.



# Fig 14 : Assumed lines of limiting stand density for Pinus caribaea



Temporary sample plots do not give very good

information for this type of analysis unless there is very clear stand history data, which is not the case here. The lower stockings observed will probably be due to a number of factors: Unstocked plots due to local heterogeneity in the forest, areas of damage from forest fire, areas of low stocking due to planting gaps, and gaps caused by felling. The self-thinning line itself is site sensitive, with a lower stocking limit on poorer site, so the solid line shown on Figure 14 represents only an upper limit.

For the purposes of modelling growth, only unthinned stands are sensitive to the selfthinning function, so these uncertainties do not greatly affect a planning model for sawlog regimes. Improved analysis requires permanent plots, so that the slope of the self-thinning curve can be estimated independently and then correlated with site index to get an empirically determined family of curves. As an interim measure, it is suggested that a stand density coefficient of 75% is likely to represent average well-managed fully stocked stands (ie. little fire, homogenous density, low planting mortality, well-weeded in the establishment phase, whilst a coefficient of 50% is probably representative of rather indifferent management.

# The yield model as an Excel workbook

The various functions described in the foregoing sections are combined into a working yield model for *Pinus caribaea* in Uganda that can be used to evaluate the effects of variable planting density and thinning regimes at different site index values.

# Workbook structure

The workbook is called *Carib Pine Yield Model.xls*. When it is opened either by doubleclicking on the file icon, or the *open* menu within Excel, it will appear as shown in Figure 15. Note that upon opening, the *Enable macros* option must be selected or the model will not work.

There should be three sheets visible, labelled *SI*, *Graph*, and *Model*. The SI sheet contains the site index curves for *Pinus caribaea*, as tabulated values and as a graph, for reference and to identify the site index of existing stands of known age and height.

	Carib Pine Yield Model															
	A	В	С	D	E	F	G	Н	1	J	K	L	М	N	0	
1				Yie	eld m	odel f	for Ca	aribbea	an Pi	ne in	Ugan	Ida				-
2	Site inde	ex	12		Plant	ing Wha	1500		Surv	vival %	85%		Density I	index %	50%	Ē.
3		Ma	in crop	before	thinnin	q			T	hinning	s		M	4	CAI	-
4	Age	Hdom	N/ha	Dg	G/ha	Volsa	Vol 1046	Thin%	N/ha	Dg	Volse	Vol 10ub	Vol <sub>sch</sub>	Vol 100	Volsa	-
5	2	1.8	1275	5.8	3.4	3	1						1.3	0.5	2.6	7
6	4	4.3	1275	9.7	9.4	17	8						4.2	2.1	7.2	
7	6	6.9	1275	12.8	16.4	48	26						8.0	4.3	15.4	
8	8	9.5	1275	15.4	23.7	95	56	50%	638	14.1	39.9	23.4	11.9	7.0	23.6	
9	10	12.0	638	19.4	18.9	95	60						13.5	8.3	20.1	
10	12	14.3	638	21.5	23.2	139	91						14.9	9.5	22.1	
11	14	16.4	638	23.3	27.3	188	127						16.3	10.8	24.5	
12	16	18.4	638	24.9	31.1	240	166						17.5	11.9	26.0	
13	18	20.2	638	26.3	34.6	294	208	0003300					18.5	12.8	26.6	
14	20	21.8	638	27.5	37.9	347	249	50%	319	25.2	145.8	104.9	19.3	13.6	26.6	
15	22	23.3	319	31.0	24.1	235	174						19.1	13.7	17.0	
16	24	24.6	319	32.0	25.7	265	198						18.8	13.6	14.9	
17	26	25.7	319	32.9	27.1	293	222						18.4	13.5	14.2	
18	28	26.8	319	33.7	28.4	320	244						18.1	13.3	13.3	-
19	30	27.7	319	34.4	29.6	345	265	50%	159	31.5	144.9	111.5	17.7	13.1	12.5	-
20	32	28.6	159	37.5	17.6	212	165						17.0	12.7	6.0	
21	34	29.3	159	38.1	18.2	224	176						16.3	12.2	6.1	
22	36	30.0	159	38.6	18.7	235	186	1					15.7	11.8	5.6	
23	38	30.6	159	39.1	19.1	246	195						15.2	11.4	5.1	
24	40	31.2	159	39.5	19.5	255	203						14.6	11.1	4.7	-
	I ≤ I ≤ Graph Model / I ≤ I ≤ Graph Model /									Þ	1					

Figure 15 : Appearance of the Carib Pine Yield Model workbook

The *Graph* sheet simply represents columns F, G, M and N of the Model sheet in graphical format, giving lines for volume to 5 cm top overbark, and 10 cm top underbark, and for MAI to the same volume limits, over time.

The *Model* sheet sets up and represents the yield model through its underlying functions and relationships, allows the various input options to be set up (site index, initial stocking, mortality, density coefficient, thinning regime), and displays all the results in tabular form as shown.

The workbook contains five macros, which appear as special worksheet functions discussed further in the next section.

# How the model works

Column A gives the age values as data. These are at either 1 or 2 year intervals. Longer intervals, with the finite difference modelling techniques employed, results in a loss in accuracy, principally in the form of underestimated volumes and MAIs.

Column B uses the special function **=***Hdom*(*SI*, *Age*) to give dominant height in metres from site index and age. Site index is entered by the user SI in cell C2, and Age will be the row value in column A. The *Hdom* function is a macro that embodies equation {14}.

Column C gives stem numbers. Initially, these are the planted number per ha from cell G2, times the survival percent from cell K2. Both of these are entered by the user. Stem numbers are checked against the self thinning line, and if they exceed the self-thinning limit, they are reduced accordingly. Stem numbers will also be reduced by thinning removals. The actual function that appears in column C has the form =*SelfThin(Nha-Nthin, Hd, zeta)*, where *Nha* is the number of stems in the previous row, *Nthin* are thinning removals in that row, *Hd* is dominant height, taken from column B, and *zeta* is the self thinning coefficient, given by the user in cell O2. The *SelfThin* macro embodies equation {25}.

Column D (mean basal area diameter) is calculated from standing volumes and stem numbers using the function =Dg(N, V), which embodies equation {24}. Basal area (column E) is then calculated from  $D_g$  conventionally as  $N.D_g^2 \pi/40000$ .

Column F (standing overbark volume to a 5 cm top) is calculated as the standing volume at the previous year, plus current annual increment from column O, less the volume removed as thinnings during the previous year.

Column G gives the underbark standing volume to a 10 cm top diameter. It is calculated from overbark standing volume (column F) and mean diameter (column D) using equation {23}, which is embodied in the function =*vub10ha*(*Vob*, *Dg*).

Column H is filled in by the user, and indicates the percentage of stems to be removed during that year as thinnings. If it is left blank, no thinning is done.

Column I is the number of stems removed as thinnings. It is calculated directly from current stocking (column C) and percentage of stems to be removed, as specified by the user (column H).

Column J is the mean basal area diameter of the thinnings. As with stand diameter, it is calculated using equation  $\{23\}$  via the function =Dg(V,N), where V is the volume thinned, and N is the number thinned.

Column K is the volume thinned, overbark to a 5 cm top. This is a proportion of the total standing volume that depends on the number of stems thinned and the standing stock. It is calculated as  $(N_{thin}/N_{stand})^{1.25}$ ; this is embodied in the function =*ThinR(Nstand, Nthin)* to allow some logical checks on the parameters. This procedure gives the effect of a moderate low thinning, with a bias in stems removed towards sizes somewhat smaller than the mean; this bias reduces as the proportion of thinned stems increases. This 'thinning ratio' method is based on that described in Alder (1979).

Column L is the volume thinned, underbark to a 10 cm top. It is calculated using the same thinning ratio method as column K, but applied to standing volume underbark to 10 cm top.

Columns M and N give mean annual increment (MAI) for overbark 5 cm and underbark 10 cm volumes respectively. Each is calculated as the total of standing volume plus accrued thinning removals divided by stand age.

Column O gives current annual increment (CAI) for volume overbark to a 5 cm top. This is central to the operation of the model as the summation of CAI gives standing volume, from which mean diameter, basal area, and underbark volume are then derived. CAI is calculated using equation {22}. In the worksheet formula, this will be seen to be done in practice by calculating standing volume for a stand of the same stocking and dominant height, via equation {21}. From this is deducted stand volume at the start of the last period. Any thinning volume during the year is added. The periodic increment is converted to an annual rate by dividing by the age difference between the current and previous row.

# User inputs

The cells marked in white on Figure 15 are user inputs. All the green cells will normally be protected and cannot be altered by the user unless the appropriate password is given with the Excel *Unprotect Sheet* menu option. The user inputs comprise site index, planting numbers, survival % of the initial planting, stand density coefficient, and the ages and intensities of thinnings. These cells are commented in Excel to give guidance to users on appropriate values. Validation tests are also applied so that if entries are not within sensible limits, an error message pops up.

# **Eucalyptus grandis**

# Sources of data

report as the PPP plots.

Growth and yield data for *Eucalyptus grandis* in Uganda has been derived from two series of permanent sample plots. The first are those established by the Uganda National Biomass Study in the plantations established under the Periurban Plantation Project between 1989 and 1994 (Drichi, 2003). These are referred to in this

The second series of plots are those established for management purposes by the Rwenzori Highlands Tea Company. These were also designed as PSPs, although many in the data set have only a first measurement available. These plots are referred to as the RHTC plots.

The PPP plots were extracted from NBS's much larger database which covers many vegetation types and mixtures of species, to include only those in *Eucalyptus grandis* plantations. This meant in practice those that fell within the Periurban plantations. This data was provided to the consultants in the form of DBF files with plot headers and tree details. These were converted into Excel files and summarised at the plot level for analysis. There were in all 107 summary records from 36 PSPs, some of which were measured three times, and some twice. The plots themselves were of a square 400 m<sup>2</sup> design. As far as possible, only planted Eucalyptus stands were included (coppice stands were excluded). The age range in the data was from 1 to 9 years, and six forests were covered: Kiwaga, Mbale, Mbarara, Namanve, Nsube, Tororo (see Figure 1). It should be noted that not all these periurban plantations remain extant, due to the rapid urban expansion in recent years, especially around Kampala. Namanve, for example has been



Nsube Cpt 9. 5-year old coppice stand of Eucalyptus grandis, marked for thinning. Originally planted 1992 under the Periurban Plantation Project, coppiced 1998.

largely re-zoned as industrial land. However, in other areas the plantations are in good order and the plots remain relocatable for further measurements. Some of these are now in their second coppice rotations (see photo above).

The RHTC plot data were provided as field forms through the company's management. The consultants arranged their data entry, checking and error correction within the context of the present assignment. The tree-level data was again summarised to give plot totals and means in a compatible format to the PPP data. The RHTC plots are circular plots of 200 m<sup>2</sup> area. In all there were 239 records, including 29 PSPs that had been measured more than once. The youngest stand was 2 years old, and the oldest was 8 years. All were planted stands (coppice not included) and located in the Fort Portal area.

A feature of both these data sets was therefore that they represented young stands, with a maximum age of 9 years. However, there are older Eucalyptus stands which could provide important data in future; and as will be discussed relative to the site index curves and volume functions, earlier studies in Uganda have included stands up to 20 years old, which usefully extend the range of information available.

The RHTC stands are generally managed to high standards, with initial fertilization and weed control, and the use of imported seed from improved sources in South Africa. The PPP stands were somewhat more *laissez-faire* in their technique, using seeds from local sources, no fertilization, and less strict weed control and re-stocking for planting mortality. This is reflected in the generally lower site index values and more variable stockings for the PPP plots versus those of RHTC.

# Tree and stand volume equations

Unlike *Pinus caribaea*, data did not exist for *Eucalyptus grandis* which the consultants were directly able to work with to revise or generate tree volume equations. However, there are a number of volume equations published for *E. grandis* and *E. saligna* (which is a very

similar species, with original plantings in Uganda from South African sources probably being a hybrid – see Marsh, 1953 and Kriek, 1968).

# Kingston's volume equation

Kingston (1972) produced an equation for *E. grandis* in Uganda as follows:

$$v = 0.00003805 - 0.00009789 d^2 + 0.0001325 dh + 0.00002967 d^2h$$
 {26}

where v is tree total volume overbark in m<sup>3</sup>, d is tree dbh in cm, and h is total height in m. Kingston's definition of 'total

volume' is not clear - whether there is a cut-off to an upper point of measurement such as 5 cm, or if he calculated a volume for a notional conical section above the last measured stem section, as is often done. His paper does however give a sample distribution by diameter and height classes, which is reproduced in Table 4. This was used as a basis for comparisons with volume equations from other countries.

Table 4 : Sample distribution for kingston's	1972 volume equation
The table shows numbers of sample trees in each size class.	The size class should be read so
that 0-3 indicates 0-3 99 etc	

						• • • • • • • • • • •				
					Heigh	nt classe	es (m)			
		0-4	5-9	10-14	15-19	20-24	25-29	29-34	35-39	40-44
	0-3	6	5							
	4-7		16	14						
	8-11		2	22	10	1				
	12-15			1	30	15	5			
Ē	16-19				10	39	14	1		
U S	20-23				5	59	16	7	1	
ass	24-27				4	27	21	21	7	
Ü	28-31					5	22	16	11	1
ter	32-35					1	5	6	8	5
ne	36-39							2	4	1
Diai	40-43							1	1	
	44-47							0	1	
	48-51							2	2	
	52-55								0	
	56-59								1	

# International comparisons

Shiver & Brister (1992) and Fonweban *et al.* (1995) produced volume equations for *E. saligna* in Kenya and Cameroon respectively. Using the sample distribution in Table 4, volumes have been calculated using their equations and Kingston's for a range of height-diameter combinations. These are plotted in Figure 16 against Kingston's equation, and also a best-fit form factor model based on Kingston's calculated volumes. It can be seen that all three are very similar, with Shiver & Brister's (1992) equation for *E. saligna* in Kenya being practically identical. The best form factor for this data distribution is 0.397. It was noted however that if only small trees are considered, as in the RHTC and PPP datasets, that the form factor tends



Fig 16 : Comparison of 3 volume equations and a simple form factor for Eucalyptus grandis/saligna

to rise as stand age decreases, being around 0.42 for the youngest stands (smallest average diameters), and typically being around 0.4 for the data sets up to 9 years old. This is very consistent with Jacovelli's (2001) recommendation based on Southern African experience of a form factor of 0.4 for fuelwood in terms of solid measure. The lower figure seen in this comparison (0.387) is due to the much larger average tree size in Kingston's sample. It is also important to bear in mind that these form factor variations are based on comparisons between volumes calculated from Kingston's equation {26} and cylindrical volume, and not between actual tree scaled volumes and cylindrical volume. Therefore too much should not be read into them, as they may only reflect the behaviour of the equation over its parameter space, not real tree form variations.

#### Volume to a 10 cm top diameter

Kingston's 1972 paper also gives top diameter conversions. However, it is apparent that these are not realistic, and are based on a misperception that would have been possible in those days of the efficiency of simple regression analysis under all circumstances. Figure 17 compares Kingston's and Shiver & Brister's (1992) conversion ratios. Kingston's model suggests for example, that 36% of the bole volume of a tree of 10 cm dbh exceeds 10 cm diameter. A tunical 10 cm *E* 

diameter. A typical 10 cm E. grandis tree is about 12-15 m high. The ratio from Kingston would imply that for a tree of exactly 10 cm at 1.3 m height, there was a point at about 4-5 m of about 10 cm diameter, with larger diameters below. This is plainly impossible. Kingston's mistake is understandable if one is experienced with the effect of transformation on the weighting of regression lines, but his model for Uganda is not usable, as it

Fig 17 : Comparison of Kingston and Shiver & Brister's merchantable volume functions



would grossly overestimate merchantable volumes in stands of smaller mean diameters. Shiver & Brister's (1992) conversion equation has therefore been used for the *Eucalyptus grandis* 10 cm calculations. The general form of their model for any top diameter limit is:

$$v_{\delta} = v (1 - 0.7426 \, \delta^{3.4138} \, .d^{-3.3125})$$
 {27}

where v is tree overbark volume,  $\delta$  is the top diameter limit in cm, d is the tree dbh, and  $v_{\delta}$  is the merchantable volume to the specified top diameter. It will be noted that this equation is of the same form as the merchantable volume equation used for *Pinus caribaea* (see equations {6}-{8} and related discussion, page 12).

#### Stand form factor and volume equation

For all the *E. grandis* plots, stand total volumes and overbark 10 cm volumes were calculated using equations {26} and {27}. The mean stand form factors was estimated at 0.327. It is defined by the equation:

$$F = V/(G.H_d)$$
<sup>{28</sup>

where F is stand form factor, V is total volume/ha, G is basal area in m2/ha, and  $H_d$  is stand dominant height.

A stand form factor of 0.33 is useful as a rule of thumb<sup>\*</sup>, but is slightly biased for the smaller mean diameter stands. A more accurate estimate of stand volume from basal area and dominant height is given by:

$$V = 0.489 (G.H_d)^{0.942}$$
 {29}





<sup>\*</sup> Tree and stand form factors should not be confused. The rule-of-thumb tree form factor for *E. grandis* is 0.4 (see page 28), while the stand form factor is 0.33. The difference is due to the fact that (a) in a stand, many trees will be shorter than the dominant height, and (b) there are average diameter distribution components hidden within the stand factor. In the same way, tree and stand volume equations should not be confused.

This relationship had an R<sup>2</sup> of 99.5% with 346 sample plots.

#### Stand merchantable volume

Figure 18 shows a graph of the ratio of volume to 10 cm top diameter over total volume, plotted against stand mean diameter. The blue points are the RHTC plots, whilst the green ones are PPP plots. The red line represents the model:

$$V_{10}/V = 1 - \exp(-0.4327(D_g - 9.5)^{0.762})$$
 {30}

This function was fitted by graphical adjustment, rather than regression. There are a number of the PPP plots which clearly have rather odd diameter distributions, with low mean basal area diameters relative to their volume to a 10 cm top. It is believed that these plots may comprise a mixture of planted stems and coppice, essentially with a multiple age structure. In the graphical analysis they are therefore ignored, and the model is aligned with the majority of plots, which conform well to the conversion function.

This equation gives overbark volume per ha to a 10 cm top, given standing total volume in  $m^3$ /ha and stand mean diameter, in cm.

#### Site index

The techniques and principles employed here for developing site index curves for *Eucalyptus grandis* are generally closely similar to those described for *Pinus caribaea* on pages 13-20, to which the reader is referred for additional details of method.

#### International comparisons

Figure 19 compares dominant height-age curves from several studies of *E. grandis*. These include Kingston (1972) for Uganda, Campos *et al* (1985) for Brazil, Saramaki & Vesa (1989) for Zambia, Shiver & Brister (1992) for Kenya, and Fonweban & Houllier (1995) for Cameroon. Superimposed on these comparisons are the PPP and RHTC permanent plot data for comparison. The RHTC plots are in magenta (pink), and the PPP plots are in green. The blue squares are plots with only a single measurement from either data set. As discussed in



Fig 19 : Median site index curves for Eucalyptus grandis from several countries

relation to the *Pinus caribaea* height growth curves (see page 13), the early asymptotes suggested by the Brazilian and Kenya curves are artefacts of their methods and data distribution, not a genuine indication of species behaviour; *Eucalyptus grandis* is a large tree which reaches 60-70 m high in its natural habitat in Australia (Anderson, 1968, p 135-137). Kingston's curve was based on older stands, and did not extend below 6 years of age; the function he used, if extrapolated below age 6, behaves very badly, and cannot be used directly for young stands. However, as later became apparent (see next section), his line accurately reflects median trends for older stands in Uganda.

It will be seen from Figure 19 that the other studies, which are based on younger stands below 10 years, agree well within that range with the Uganda data. It is also notable that the RHTC plots show generally better height growth than the PPP plots; it will be recalled that the former receive fertilizer treatment at planting, are based on improved seed, and are probably better weeded in their early years. These factors most probably obscure any environmental differences between planting areas.

#### Kriek's data for Uganda Research Plots

At a later stage in this study, after the site index curves described below had been fitted, a report by Kriek (1969) came to light which gives in graphical form what was possibly the source data for Kingston's curves. This shows height growth for a number of research plots (RPs), mainly *E. grandis*, but also including trials of *E. citriodora*, *E. paniculata*, *E. cloeziana*, and *E. deglupta*. His graphical data has been 'reverse engineered' by the consultants (using a ruler) to extract the growth data in digital form for *E. grandis* only. The results are reproduced in Figure 20 below. This data have been used to validate the completed curves, as discussed in the next section, although they did not contribute to their actual development due to their late emergence in the course of the study.





In the graph, the line numbers used are the same as those in Kriek's (1969) report, for ease of cross-reference. The RP plot numbers and locations are shown in the table<sup>\*</sup>. Kingston's 1972 median site index line is shown in red for comparison. At least some of these plots are still extant, and it is an important recommendation of this report that they should be protected

<sup>\*</sup> However, RP 420 in the table is an error. It should read RP 429.

and re-measured as soon as possible. They are an important resource, both for information, as demonstration plots, and as a source of seed of known performance.

### Fitting of site index curves

Site index curves were fitted to the data using the same graphical methods that have been described for *Pinus caribaea* (see page 15). Figure 21 shows the Excel graphical tool used for this purpose. The PPP (green) and RHTC (magenta) data sets are shown, together with Kingston's median curve (blue). The red lines show the site index model, with the control points for calibrating each model as green diamonds. The median, lower and upper curves were adjusted independently to give a good visual fit to the tendency of the data. It was found during this process that the Chapman-Richards function, which had given the best result for *Pinus caribaea*, did not perform well, tending to flatten off too much when the lower part of the curve was correctly adjusted. The curve was changed to the Schumacher model, which performed better for this data set. It is notable that in Figure 19, the Kenyan and Brazilian curves, both of which show early asymptotes, use the Chapman-Richards function, whereas the Zambian and Cameroon equations, with more realistic later height growth, are based on the Schumacher curve.



Fig 21 : Site index fitting tool for E. grandis in Excel

After fitting the curves by eye, the coefficients were regressed against site index (using a base age of 10 years as for *Pinus caribaea*) to give a general model, which has the form:



Here  $H_d$  is stand dominant height in m, t is age in years, S is site index, or dominant height at age 10, and A, b, and k are coefficients.

# Validation of the curves with Kriek's data

Figure 22 plots the curves from equation {31} with site index values from 22 to 34 m, which covers the range of height growth for the PPP and RHTC plots. Kriek's (1969) data from Figure 20 is superimposed as green lines. It can be seen that the fit is excellent. In addition, there may be a higher site class which should be represented, with curves up to 38 m at age 10.



Fig 22 : Site index curves for *Eucalyptus grandis* Kriek's (1969) Research Plot data shown as green lines. These data were not used in fitting the curves.

	As discussed for Pinus caribaea, pages 20-23, the
Yield model based on	dynamic yield model has at its core a predictive model
dominant height and stocking	for stand volume based on an assumption of constant
	spacing. For Eucalyptus grandis, a function was fitted
	directly with the form:

$$V = 0.008429 \quad (H_d - 2.5)^{2.148} N^{0.4933}$$
(32)

Here,  $H_d$  is dominant height in m, N is stocking in trees/ha, and V is total volume (overbark to 5 cm top). This equation fitted with an R<sup>2</sup> of 95.9% and 346 data points (plot records). Figure 23 shows the form of this curve for different constant mean spacings. On the figure the blue squares are the RHTC plots, and the green ones the PPP plots.



# Self-thinning

The reader is referred to page 23 for background literature on the general topic of self-thinning, where it is discussed in the context of the *Pinus caribaea* yield model. Generally because *E. grandis* has been planted at

higher densities for maximum fuelwood production, and because the data is from permanent plots, rather than temporary plots as for *Pinus caribaea*, the self-thinning function is more clearly apparent. Against this, it must be said that E. grandis has a reputation for self-thinning even before crown contact (Bredenkamp & Burkhart, 1990), which means that there is not necessarily a single well-defined Reineke Line at the self-thinning limit (Reineke, 1933), but rather a fuzzy region that is somewhat site and silvicultural treatment dependent.

The figures overleaf show the self-thinning limit using the conventional Reineke scales of N/ha on Dg, and the more practically useful N/ha on dominant height.



**Fig 24 : Stand density diagram for Eucalyptus grandis plots** RHTC plots are pink squares. PPP plots are blue circles. Successive measurements of PSPs are joined by grey lines. The Reineke self-thinning limit is shown in red. A line of constant basal area (35 m²/ha) is shown in yellow. The green line shows the limit of stand closure.

Fig 25 : Self-thinning as a function of dominant height for *Eucalyptus grandis* RHTC plots are shown in pink and PPP plots in green. The self-thinning limit is shown in red.



It can be seen that there are many plots that lie below the self-thinning limit but yet show stocking declines between measurements. However, most of the RHTC plots, with their more strictly controlled treatment, are close to the self-thinning line. The lower stockings of the PPP plots need to be investigated to improve the model, but could be due to planting failures, weed competition, or partial harvesting.

#### Self-thinning function for the yield model

For the yield model, the same self-thinning concept is proposed as for *Pinus caribaea* (see page 24, with a limiting stocking relative to dominant height given by the red line on Figure 25, which is represented by the equation:

N= 
$$396460 H_d^{-1.6066} \zeta$$
 {25

where  $\zeta$  is 1 for the limiting line, or has values such as 0.75 to represent typical good quality management, 0.50 for laissez-faire management, and 0.25 for very poorly stocked stands with high mortality for whatever reason (see also Figure 14, page 24).

#### Further work on self-thinning and stand density

There is a wealth of information on responses of *E. grandis* to spacing from studies from southern Africa and elsewhere (*eg.* van Laar & Bredenkamp, 1979; Bredenkamp, 1982; Bredenkamp & Gregoire, 1988; Merriam *et al*, 1995; Lei Yuancai *et al*, 1997; Mabvurira & Pukkala, 2002; Mabvurira *et al*, 2002; Mabvurira & Miina, 2002, amongst others). Kriek's (1969) study in Uganda also contains important results on spacing responses, although the quality of the surviving diagrams requires examination of almost archaeological or forensic intensity to extract the most useful data. The authors have not had time within the context of this short study to go into sufficient detail on this matter, but it would be a useful desk exercise to consider for the future to improve this aspect of the model based on the published literature. In its present form, the model under-represents responses to wider spacings, and is therefore conservative.

# The Excel yield model for *Eucalyptus grandis*

An Excel workbook called *E.grandis Yield Model.xls* has been created that integrates the foregoing analyses and equations into a single dynamic yield model. This is identical in structure and design to the model for *Pinus* 

*caribaea* as described in some detail on pages 24-26, so this detailed description and user information will not be repeated here. Figure 26 shows the appearance of the model, with the white areas being fields that the user can modify to simulate different sites and management regimes; the green areas give calculated results, but are password-protected against direct user manipulation.

	Yield model for <i>Eucalyptus grandis</i> in Uganda													
Site inde	ex	34		Planti	ng N/ha	2200		Surv	ival %	85%		Density i	ndex %	75%
	М	lain crop	before t	thinning				Thinnings			MA	AI .	CAI	
Age	Hdom	N/ha	Dg	G/ha	Vol <sub>5ob</sub>	Vol <sub>10ub</sub>	Thin%	N/ha	Dg	Vol <sub>5ob</sub>	Vol <sub>10ub</sub>	Vol <sub>5ob</sub>	Vol <sub>10ub</sub>	Vol <sub>5ob</sub>
1	8.0	1870	5.4	4.2	13							13.5		13.5
2	14.3	1870	9.6	13.5	70	5						34.8	2.5	56.1
3	18.8	1870	12.1	21.5	139	82						46.3	27.4	69.4
4	22.2	1870	13.8	28.1	210	154						52.5	38.5	71.0
5	25.0	1685	15.5	32.0	265	216						53.0	43.3	54.9
6	27.4	1461	17.2	34.0	305	265						50.8	44.2	40.0
7	29.3	1304	18.6	35.6	340	307						48.6	43.9	35.4
8	31.1	1189	19.9	36.9	372	343						46.5	42.9	31.7
9	32.6	1101	21.0	38.1	401	376						44.5	41.7	28.7
10	34.0	1030	22.0	39.1	427	404						42.7	40.4	26.1
12	36.4	924	23.7	40.7	473	455						39.4	37.9	23.1
14	38.4	848	25.1	42.1	513	497						36.6	35.5	19.8
16	40.1	791	26.4	43.2	547	534						34.2	33.4	17.4
18	41.6	746	27.4	44.1	578	566						32.1	31.5	15.4
20	42.9	709	28.4	44.9	606	595						30.3	29.8	13.8
22	44.1	679	29.3	45.6	631	621						28.7	28.2	12.5
24	45.2	653	30.0	46.3	654	645						27.2	26.9	11.4
26	46.1	631	30.7	46.8	675	667						26.0	25.6	10.5
28	47.0	612	31.4	47.4	694	687						24.8	24.5	9.7
30	47.8	595	32.0	47.8	712	705						23.7	23.5	9.0

Fig 26 :	The Eucalyptus	grandis Excel	yield model,	with simulation	of typical RHTC	stands
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# A reality check

Figure 26 has been set up with very typical parameters for the RHTC plots, including a site index of 34 m at 10 years, stocking after initial mortality of around 1870 trees/ha, and unthinned. The MAI for 10 cm volume overbark is shown on Figure 27 overlaid on actual plot data. It can be seen that the shape and position of the curve indeed conforms to typical behaviour for the plots.

The plots show a wide spread of results. To investigate these variations requires first that plots are averaged at the compartment level, as individual plots of 200 m2 will inevitably show substantial variations due to sampling alone that do not truly reflect differences in overall performance.



Fig 27 : MAI projections from the model overlaid on RHTC plot data

# Yield of Caribbean Pine

The Excel-based yield model that has been developed during this study (see page 24) allows the influence of site index, planting stockings, survival, and thinning to be explored. Typically however, average sites for Pinus

caribaea in Uganda have a site index of 12-14 m (dominant height at age 10). If planted at 1100 stems/ha ( $3 \times 3 \text{ m spacing}$ ), with 85% survival, and unthinned, such stands will have a maximum Mean Annual Increment from 19-23 m<sup>3</sup>/ha/yr occurring at ages 22-26 years.

Thinning will result in larger average size stems with better recovery on milling, but overall, thinning will tend to reduce yields per hectare and delay the age of maximum MAI. With the same range of site index (12-14 m), planted at currently recommended 1320 trees/ha (2.7 x 2.7 m), 85% survival, and silviculturally thinned by removing one tree in three (33%) at age 4, MAI of the final crop will be reduced to 17-21 m<sup>3</sup>/ha/yr at ages 26-32.



However, proper analysis of the benefits of thinning, as well as from site selection and early management, is an economic matter. Thinning will reduce the number of stems in the final crop, but increase their average size and improve their quality, significantly improving recovered volume. If for example, as a result of thinning, recovered lumber volume increases from 25% to 33%<sup>\*</sup>, there may be a substantial economic benefit even with lower gross volumes. Thinnings can also give intermediate returns, although these are likely to be small.

The model does reveal the sensitivity of production to two important management factors: Early survival, and site index. Both of these depend on good site selection, good seed selection, good nursery technique, good planting technique, and careful tending during the first two years. These factors will influence both height growth (site index) and survival, and

<sup>\*</sup> Unless a sawmill is part of an integrated complex producing comminuted wood products, the author's experience suggests that the absolute maximum possible recovery from gross volume to lumber, with highly efficient re-sawing, will be 45%. More realistically, single line band mills will yield 30-35%, and circular saws perhaps 25-30%. under efficient management. Recovery is sensitive to tree size and market requirements. The size distribution of trees, which is not available from the present yield models, gives therefore important technical and economic information.

as the yield model demonstrates, have a strong and sensitive influence on final yield, and hence on profitability. Figure 28 shows how for unthinned stands, maximum MAI increases from 17-26  $m^3/ha/yr$ , and rotation decreases from 33 to 22 years as site index increases from 12 to 14 m.

It will be noted that existing stands typify the results of rather *laissez faire* management, with poor form due probably to the use of locally collected seed, rather than improved seed. This study has not considered the recovery rates possible for existing or future stands. There is information in the FRP database on stem form, but actual recovery depends very much on the technical sawmilling systems in use.

Table 3 on page 19 shows *Pinus caribaea*'s performance on some different sites. It has been widely planted around Katugo and Kikonda. These tend towards lower average site indices of around 12-13 m. Better sites occur at higher altitude and further west, where growth may be around 14-15 m.

As a general summary, it can be said that on average sites but with good management, *Pinus caribaea* has a yield of about 22 m<sup>3</sup>/ha/yr of underbark volume to a 10 cm top diameter on a rotation of about 25 years.

# Yield of Eucalyptus grandis

*Eucalyptus grandis* shows more rapid growth in terms of height and diameter than Pinus caribaea. The height growth curves are shown in Figure 22 (page 34). At age

10, *E. grandis* will attain stand dominant heights of 22 - 36 m, compared with *P. caribaea's* 8 - 20m. This height growth will continue, so that on the best sites, dominant heights may be 45 m at age 20.

The yield model for E. grandis which is described in this study is contained in a workbook called *E. grandis Yield Model.xls*. This allows yields to be predicted under various conditions of site index, initial planting density and thinning. Figure 29 shows the mean annual increment for unthinned Eucalyptus on the good, median and poor sites, assuming planting at a moderate density of 1660 trees/ha (3 x2 m), and with 85% survival. The volume increment is to a 10 cm top diameter, overbark.

It can be seen that on the good sites, yields of better then 45 m<sup>3</sup>/ha yr will be achieved on a 6 vear rotation. On median sites, yields will be around  $32 \text{ m}^3/\text{ha}/\text{yr}$ on a 9 year rotation, whilst on poorer sites, optimum production will be around 21  $m^3/ha/yr$  on a 13 year rotation. 'Site' in this context implies any factor which can affect site index, and may include, as well as the natural suitability of





allocation in terms of soil and climate, forest management factors such as selection of improved seed, appropriate fetilization, early weed control, and fire control. It may be noted

from figure 21 on page 33 that the RHTC plot data were all above the median site index, whilst the PPP plots, which varied more in both locational factors and management history, represented both good and bad site index values.

#### **Diameter response to thinning**

The yield model is conservative in terms of diameter responses to thinning. The empirical data available does not directly provide good information on spacing effects, mainly because most of the plots were from unthinned stands at high initial stockings. This can be seen in

Figure 30, which compares unthinned stands with two thinning regimes. The unthinned stand assumes initial stocking of 2200 stems; the thinned stands assume initial stocking of 1660 stems with, respectively, two thinnings of 66%, or three thinnings of 50%. In each case the model shows a strong diameter response to the selection effect of a low thinning, but the growth response to wider spacings is negligible.





This aspect of the model may not be fully realistic, but could be improved in future with a little further work based on available information in the scientific literature. This would make the model more useful in choosing between different thinning regimes, as mean diameter has a strong influence on the economics of sawlog or veneer production.

General monitoring recommendations for plantation management As has been noted in this study, there exist monitoring plots within both the Periurban Plantation Project areas, and the Rwenzori Highlands Tea Company forests. These two dispositions have different designs and objectives. The PPP plots are square plots of 20 x 20 m (400 m<sup>2</sup>), established as part of the national biomass Study. They are designed primarily for assessment of

biomass and biomass productivity. The RHTC plots are 8 m radius circular plots (201 m<sup>2</sup> area), and are intended both to provide forest growth and yield data, and as a system of continuous forest inventory.

Neither system is ideal for routine forest management. Square plots are useful as a standard across varied vegetation types, but in plantations may be biased<sup>\*</sup> and are rather slow to demarcate. The circular plots of RHTC are too small to provide reliable data if the stand is thinned, coppiced or managed beyond the fuelwood rotation.

# Plot design, demarcation and measurement procedure

The consultant recommend for future use circular plots of 12 m radius (452 m<sup>2</sup>). These conform to designs used in the 1960's in East Africa, and to common plantation and inventory PSP standards in a number of other countries (Adlard, 1990).

<sup>\*</sup> Philip (1983) page 218, Fig 47, illustrates how a square plot, depending on its placement relative to rows, may give over a 100% difference in measured growing stock on the same stand.

On these PSPs, the following demarcation procedures and measurements should be carried out:

- □ The plot centre should be permanently marked, either by using a cross shaped trench 40 cm deep, 40 cm wide, and 120 cm long in each direction, with the intersection at the exact plot centre; or by using an 8 cm diameter concrete pillar 40 cm long, with a piece of embedded reinforcing rod, placed at the plot centre and buried 30 cm deep.
- □ All trees on the plot should have the point of measurement (normally at 1.3 m except for deformed or forked trees) painted with a standard colour (bright red is recommended) entirely around the tree in a 2.5 cm wide band.
- Edge tree should be identified and included or excluded with great care. If the centre of the tree exceeds exactly 12.0 m from the plot centre, the tree should be excluded. On steep slopes, a correction factor should be applied for the slope % as measured along the line of measurement (which may not be the steepest direction up the slope). The slope can also be compensated for by stepping the tape. The table at the right shows the distances in metres to be measured to edge trees, depending on the slope in % or in degrees.

Critical distances equivalent to 12 m					
Slope <sup>o</sup>	Distance				
0.00	12.00				
2.86	12.01				
5.71	12.06				
8.53	12.13				
11.31	12.24				
14.04	12.37				
16.70	12.53				
19.29	12.71				
21.80	12.92				
	nces equiva Slope ° 0.00 2.86 5.71 8.53 11.31 14.04 16.70 19.29 21.80				

- □ Trees should be marked with metal tags and nails. Nails specifically for PSP work are recommended. These should have spiral shanks, making them difficult to remove, be of non-corroding alloy, have high stiffness to penetrate harder wood, but low shear strength to minimize damage to saws. Write on or punched tags may be used, but trees should be number systematically. Starting at 1, working clockwise from due north is recommended. The tag should be placed exactly 30 cm above the point of measurement, on the north side of the tree.
- □ For multiple or coppice stems, number tree in the form 10a, 10b etc. A multiple stem is one which forks below 1.3 m. In this case, each stem is measured as a separate tree, but the number should link them. Likewise coppice stems on the same stool should share a number, and be distinguished by a letter.
- □ The bearing and distance to the plot centre from the three most central trees should also be recorded as a precaution against the loss of the centre mark. This is however, not a substitute for properly marking the plot centre.
- **□** Each tree should be measured for diameter at the exact point of measurement.
- □ If the species differs from the main plantation species recorded for the plot, it should be recorded. A system of

recorded. A system of
species code letters should
be adopted for this
purpose.

- Trees should be scored for stem straightness and quality, on the scale shown at the right.
- The four largest diameter trees on the plot should be selected for height measurement. The height of each tree should be measured and recorded twice, from opposite sides.

1	1	Stem severely defective, decayed, bent, forked, with no usable timber currently or potentially.
	2	Stem has severe defect, low forking, contains major bends, but could potentially contain a single section of sawlog.
	3	Stem generally lacks straightness, several curves, sweeps, etc, one or more defects including high forks, but at least 50% of stem volume judged potentially usable for saw timber.
	4	A single minor defect (small rotten branch etc.), minor curvature or sweep on one section of stem not exceeding half the stem diameter in deviation from straightness.
	5	Stem defect-free and perfectly straight.

On slopes, measurements should be made at right angles to the slope. Trees of large diameter but with crown damage (broken tops, etc.) should not be included in this sample.

The tree should be assessed qualitatively for various abnormalities and damage, and a coded note from the following list applied if possible. If there is no suitable coded note, an explanatory comment can be written.

Coded note	Description	Action required.					
Dead or missing trees							
NT	No tree. A tree present at an earlier measurement cannot be found at all (no stump, fallen stem or other trace).	This code <u>must</u> be recorded for any missing tree that cannot be found at all.					
DT	Dead tree. Tree clearly dead, but still standing. Suppressed tree with no live foliage.	Standing dead trees must be measured for normally diameter. On PSPs it is not unusual for them to come back to life at next measurement.					
FT	Fallen tree. Tree fallen and on the ground. The tree may still be alive. The root system has not been uplifted (see UT)	It is not necessary to measure such trees. A tree is fallen if the angle with the ground is less than $45^{\circ}$ other wise use LT code. Combine with DT code if tree is dead.					
HT	Harvested tree. A cut stump has been found, indicating a harvested tree.						
UT	Uprooted tree (probably windthrow). The tree has fallen with the root system being exposed.	Assess as for FT – the only difference is that the roots have been uplifted, almost certainly indicating wind damage.					
Leaning or dam	aged trees, defect						
LT	Leaning tree. The tree is leaning, but at an angle of less than 45° from the vertical, has not been fully uprooted, and is still alive.	These trees should be measured for diameter.					
FD	Fire damage. Charred bark, burnt pipe or branches, or burnt foliage from fire.						
RS	Rotten stem. Signs of fungi or rot on the bole, rotten pipe at base of tree.						
RB	Rotten branch. Dead or rotten branch, decay in the upper stem or crown.						
EX	Excresences. Bumps or growths on the stem, sometimes with epicormic growth. Usually a sign of fungal or insect damage.						
EB	Epicormic branch growth. Usually a sign of ill-health or past severe fire damage.						
AD	Ants or termite damage. Evident signs of ant or termite damage (hollowed bole, small trees can be pushed over).	Do not use this code if ants/termites are present but there is no direct evidence of tree damage.					
BT	Broken top – main bole broken.	Look for signs of decay and add RS code if found (but not if unsure).					
CD	Crown damage. Branches or tip of crown damaged for any reason (wind, tree felling etc.).	Do not use this code for dieback.					
DB	Dieback. Leading branches are dead, but main crown still alive.	Do not use this for foliage loss due to suppression or firedamage.					
FS	Forked stem.	Make a note of the estimated height of the fork. If the stem forks at or below 1.3 m, the tree should be counted as two trees on the plot, with the MS code.					
FX	Foxtail. Abnormally long, branchless leader, particularly applicable to <i>P. caribaea</i> and <i>P. oocarpa</i> .						
Regeneration ar	Regeneration and silviculture						
NR	Natural regeneration. (do not apply to coppice).	Only include if stem exceeds 5 cm dbh. Make sure species recorded if not the same as the main crop.					
СР	Coppice.	This must always be given at the first measurement for all coppice stems.					

# Number and location of plots

The number, location and sampling intensity of PSPs depends on whether they are being used solely for growth and yield (G&Y) studies, or also as a system of Continuous Forest Inventory (CFI). A well managed plantation enterprise will tend to rely on some form of CFI as part of its management information system (MIS), simply on grounds of efficiency. However, the design of an appropriate CFI system depends on many factors that are outside the scope of the consultants control or determination within the context of this study, so the issue of plot numbers and location will be considered here solely from a G&Y perspective.

It is recommended that PSPs according to the above standards are established:

- Within existing research plots that can be located and protected. Some of these plots, such as RP429 outside Fort Portal (photo), are extremely valuable resources from three points of view (a) to demonstrate forestry potential to investors, (b) as a source of improved seed, and (c) for growth and yield information.
- □ In stands which are scheduled to be thinned, to obtain information on thinning responses.
- □ In new plantings, on a 250 x 200m grid (approximately 1 plot per 5 ha). This should be done only from the second year after planting, and PSPs should only be established which can be properly measured and maintained from a recurrent budget allocated for this purpose. A grid design cannot always be applied in irregular shaped compartments. The objective should be to establish PSPs as uniformly distributed as possible, avoiding locations within 50 m of forest edges or roads, and abnormal areas in terms of stocking, drainage or topography.

PSPs of this design should not replace the existing NBS plots, and it is also recommended that high priority is given to remeasuring the NBS plots that exist within the extant periurban plantations. This remeasurement of the PPP plots should also collect a brief narrative history about the management of each plot, especially since the last measurement some 5 years ago.

RHTC could also follow the above recommendations. The authors would recommend that they site some of the larger plots concentrically on their existing plots, keeping the same tree numbers for the original trees.

Remeasurements of these PSPs should be made every two years, with the plots being established two years after planting. At the initial establishment, the plots should be georeferenced using a GPS, which should at that stage be straightforward, as the canopy will still be quite low. Remeasurements should also be made immediately before thinning and final harvest. When the stand is regenerated by coppice, the plot should be remeasured two years after coppicing.

# Human resources and data processing

The consultants recommend that PSPs are demarcated and measured by specialist teams, although some local labour can be used for cleaning, weeding and porterage. All aspects of PSP work are skilled, including painting and numbering of trees, tree measurement, and especially the supervisory and recording roles. These skills cannot be learnt overnight, but require experience to be properly bedded in. It is therefore inefficient and unsatisfactory to attempt to use inexperienced staff for these duties, and typically results in a great waste of resources as the work is done badly and yields data that is not useable.

It is suggested that the NFA should consider the following HR arrangements for G&Y work: A senior officer, who may be termed Growth & Yield Officer, Forest Biometrician, etc, with responsibility for planning, directing, and managing all inventory, PSP and mensurational

work. This officer would also process data and produce reports, analyses and publications from them.

There would be a single office assistant, mainly undertaking data entry and editing duties.

For plantation PSPs, there would be one or more field teams, comprising a forester (either degree or college trained) who would control field work and book it. This person would also assist with data entry. They would be supported by 3 rangers or skilled (ie. trained on the job) labourers who would do demarcation, and measurement work.

A single team could be expected to manage about 600 plantation PSPs on a 2-year measurement cycle (ie. 300 PSPs per year).

Data processing needs to be planned at the outset, with proper provision for equipment (computers) and software. The latter will inevitably be bespoke, and it is desirable that the facilities of the current project are used to develop and test suitable software for entering and processing plantation PSP data. If this is done using an open source approach with Excel, then the NFA should be able to take forward the evolution of the software as its needs develop, using in-house personnel.

# PSP programs and private sector plantings

The authors strongly advise against making the establishment and measurement of PSPs a statutory or contractual requirement. Efforts in this direction have been made in many countries (*eg.* Indonesia, Philippines, Brazil, etc.) and inevitably produce no useable information. Although there are many private sector companies around the world who operate highly effective G&Y research and PSP systems (RHTC is a good example in Uganda), the incentive to do this is invariably internal, and the companies involved tend to be those with a deep technical base. There can be mutual benefits from sharing research skills and facilities between the private and public sector, as has been realised with RHTC, but this can only be effective if it is done on a voluntary basis, and with a clear appreciation of mutual advantage.

Priority needs for work on plantation growth and yield

There are several areas where additional or supplementary field work and data would allow the yield models developed in this study to be extended. These include the following:

- Additional RHTC data. The PSPs processed for RHTC comprised only a subset of their data. From discussions arising during the presentations, it was learnt that there are other plots on possibly less favourable sites.
- Establishment of PSPs in the Research Plots that can be relocated. It is suggested that anumber of PSP should be established in the existing RPs. In particular, RP 429 at Fort Portal should be given first priority; its protection should also be urgently discussed with local government and other stakeholders.
- Remeasurement of identifiable PPP plots, together with a detailed management history of each plot (initial espacement, thinning, fellings, and other events).
- □ *Further work on the existing yield models,* to incorporate this additional data, improve their sensitivity to spacing and thinning effects on diameter increment, and add some economic analysis columns to the models.
- Provision of yield models for other species. There is data on Pinus oocarpa, Pinus patula, and Cupressus lusitanica. There are already good models for the latter two species (see Alder,

1979), which only need to be converted into an appropriate modern format for ease of use. Provisional models could also be developed for *Maesopsis eminii* and *Araucaria hunsteinii* (syn. *A. klinkii*), for example, if time is allowed for directed field sampling.

# Summary of outputs from this study

The present study has produced as outputs flexible yield models for *Pinus caribaea* and *Eucalyptus grandis* that can be used for decision making and financial analysis. These models, in the form of Excel workbooks, had on many 24 and 27 of this report

are simple and easy to use, as described on pages 24 and 37 of this report.

As an intermediate output that also provides a resource for further study, the softwood plantation data from the 1989-93 inventory has been converted into an Excel file, with the original raw data and plot summaries. This includes a number of other species apart from *Pinus caribaea*, notably *Pinus oocarpa*, *Pinus patula*, and *Cupressus lusitanica*.

The plot summaries for *Eucalyptus grandis* are also provided as a file.

Two presentations have been given, at Kiko Tea Estate on 16th April, and at the Forestry Department on 23rd April. These are available as power point slides. They cover much of the material in this report in presentation format.

This report and its technical content also constitutes an output from the study. These various files have been placed at a download point on the Internet from which they can be retrieved, and supplied on CD-ROM. Any queries about these various files can be directed to the first author at <u>denis@bio-met.co.uk</u>.

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<u>uymppt1.zip</u>	180	Growth and yield of Eucalyptus grandis in Uganda.ppt				
uymppt2.zip	697	Yield of E.grandis & Carib Pine in Uganda.ppt				

# Conclusions

This study has analysed existing data for two key plantation species in Uganda, *Pinus caribaea* and *Eucalyptus grandis*, and used it to provide growth and yield models for each species that can be used for

planning. These models are provided as simple Excel workbooks for further application. This report contains all the technical background, references and information required regarding these models.

Given the limited time allocated for this task, there are possibilities for improving the models with further field sampling and technical analysis. This is particularly necessary if they are to be used for coppice stands of *E. grandis* and for the economic analysis of silvicultural regimes designed to maximize production of large dimension timber. The models also do not address the issue of wood quality or recovery rates, which can depend on many silvicultural and technical factors.

It is also desirable to extend the scope of available yield models to cover other key species.

General recommendations have been given on appropriate designs for growth and yield monitoring plots. However, there are a number of institutional specifics which need to be considered relative to inventory requirements and management information. These are outside the scope of this report, which should not therefore be regarded as definitive in this respect.

The authors hope that this report may contribute usefully to the further development of a healthy and economically important plantation sector in the context of Ugandan forestry.

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